

LoPSTer Class-E PA design sheet

This sheet calculates the components for a class-E PA used on LoPSTer.

The class-E design equations are from "N O Sokal, IEEE MTT-S Digest, Class E switching mode high efficiency PA, Improved Design Equations", 2000.

Other equations are either from Pozar "Microwave Engineering", or the Circuit Sage website. The conversion from ABCD to S21 is done with reference to Dean Frickley, MTT Feb 1994, "Conversion between S and ABCD valid for complex impedances"

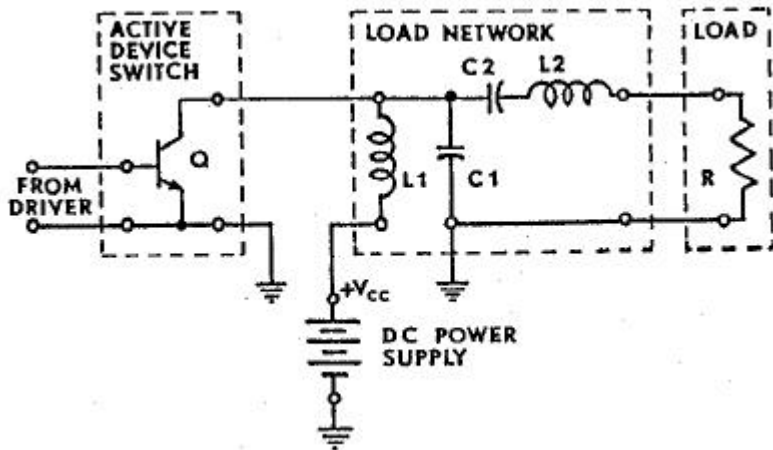
Use the Design Procedure section on page 4.

You then have a choice of four matching networks for $R_{load} < 50$ ohms (matching 1 to 4) and four matching networks for $R_{load} > 50$ ohms (matching 5 to 8). The component values, output power & harmonics are calculated for each of the circuit topologies.

Chris Haji-Michael

Inputs yellow, outputs green

$$mW \equiv \frac{W}{1000} \quad nH \equiv \frac{\mu H}{1000}$$



434MHz design sheet 10dBm, on the Reference Board

First Calculate C1

C1_total is the total capacitance at C1 and is made up of several capacitance. It includes the PCB capacitance measured using an impedance analyser on a blank board. Here, this is **multiplied by 1.4** to allow for extra stray capacitance when components are soldered. (refer: chap_12_general_information/Customersupport/2008-01-04,Fuba_RX_868MHz.pdf). Add to this the LoPSTer output capacitance, C1 and the capacitance for L1.

Finally, L1 backs-off this capacitance to give an effective C1 (C1_eff) which is used in the calculation for load impedance. This adjustment comes from Sokal and in his equations was added to give C1 which here I have called C1_total. Because I am starting with C1_total and doing things in reverse his adjustment is subtracted to give C1_eff used in his equations.

$$C_{L1} := \frac{1}{L1 \cdot (2 \cdot \pi \cdot SRFL1)^2}$$

$$C1_{total} := C_{pcb1} \cdot 1.4 + C_{lopster} + C_{L1} + C1$$

$$C_{L1} = 0.069 \text{ pF}$$

$$Zc := \frac{1}{2 \cdot \pi \cdot \text{freq} \cdot C1_{total}}$$

$$L1_{suggest} := \frac{4 \cdot Zc}{2 \cdot \pi \cdot \text{freq}}$$

$$C1_{total} = 2.651 \text{ pF}$$

$$C1_{eff} := C1_{total} - \frac{0.7}{(2 \cdot \pi \cdot \text{freq})^2 \cdot L1}$$

$$C1_{eff} = 2.128 \text{ pF}$$

$$Zc = 138.341 \text{ ohm}$$

Calculate Vo and ON-resistance

Some of this comes from the Sokal paper. I_{cc} is the average current taken in the drain of the switching transistors and causes a voltage drop across the inductor L1 and the transistor R and L. The voltage across the FET on-resistance is used to calculate V_o as $2 \cdot I_{cc} \cdot R_{on}$. The factor of 2 is used because the transistor is only switched-on half the time. The voltage drop across inductor L1 is calculated with $1 \cdot I_{cc}$.

$$I_{cc} := \frac{P_{out_guess}}{V_{cc} \cdot Eff} \quad V_{cc_eff} := V_{cc} - ESRL1 \cdot I_{cc} \quad Z_{Tran_L} := 2 \cdot \pi \cdot freq \cdot Tran_L$$

$$V_{Lon} := Z_{Tran_L} \cdot I_{cc} \cdot 2 \quad V_{Ron} := Tran_R \cdot I_{cc} \cdot 2 \quad V_o := V_{Lon} + V_{Ron} \quad V_o \text{ is the saturation voltage}$$

$$Tran_R_proposed(V_{Ron}) := \left[12.4985 \cdot \left(\frac{V_{Ron}}{V} \right)^2 - 0.136113 \cdot \frac{V_{Ron}}{V} + 9.42926 \right] \text{ohm}$$

From simulations of RON with drain-voltage under dc conditions. This is with ACOM=31.

$$V_{cc_eff} = 1.488 \text{ V}$$

$$V_{Ron} = 0.212 \text{ V}$$

$$V_o = 0.247 \text{ V}$$

$$Tran_R_proposed(V_{Ron}) = 9.961 \text{ ohm}$$

Calculate the LoPSTer capacitance

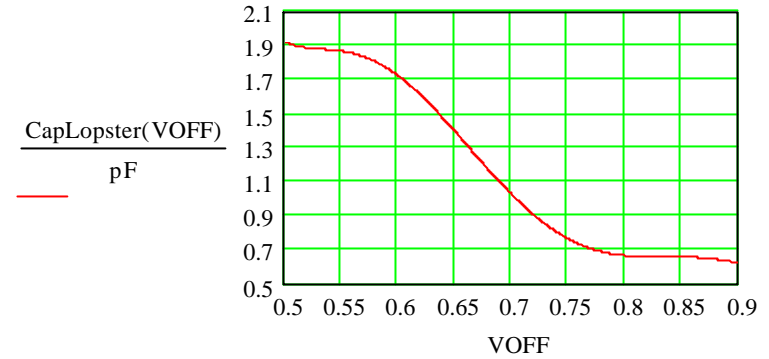
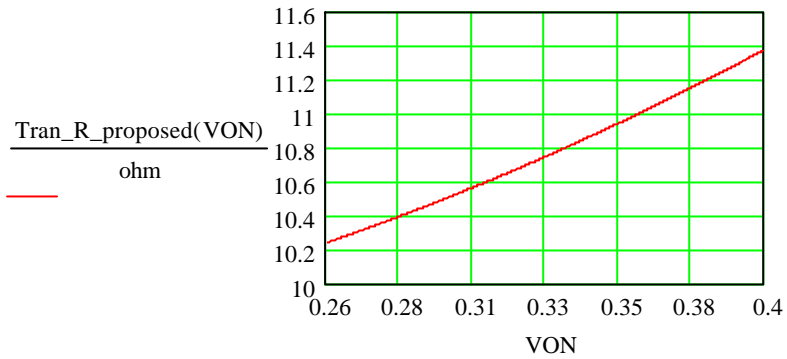
The lopster capacitance varies greatly with drain voltage. The capacitance is important to know at the drain voltage when the transistor switches-on (V_{off}).

$$V_o = 0.247 \text{ V} \quad V_{peak} := 2 \cdot V_{cc_eff} \quad V_{off} := V_o + (V_{peak} - V_o) \cdot 0.2$$

$$CapLopster(V_{off}) := \left[8627.9 \cdot \left(\frac{V_{off}}{V} \right)^6 - 38102 \cdot \left(\frac{V_{off}}{V} \right)^5 + 69153.1 \cdot \left(\frac{V_{off}}{V} \right)^4 - 65955.2 \cdot \left(\frac{V_{off}}{V} \right)^3 + 34836.2 \cdot \left(\frac{V_{off}}{V} \right)^2 - 9660.4 \cdot \left(\frac{V_{off}}{V} \right) + 1101.28 \right] \cdot \text{pF}$$

$$V_{off} = 0.792 \text{ V}$$

$$CapLopster(V_{off}) = 0.67 \text{ pF}$$



Regulator Output Voltage

$$VCC_REG(Icc) := \left[-7.6 \cdot 10^{-7} \left(\frac{Icc}{mA} \right)^3 + 4.8 \cdot 10^{-5} \left(\frac{Icc}{mA} \right)^2 - 0.00164 \left(\frac{Icc}{mA} \right) + 1.5219 \right] V$$

This is the PA regulator output voltage for PAM0 which varies with load

$$Icc = 10.632 \text{ mA}$$

$$VCC_REG(Icc) = 1.509 \text{ V}$$

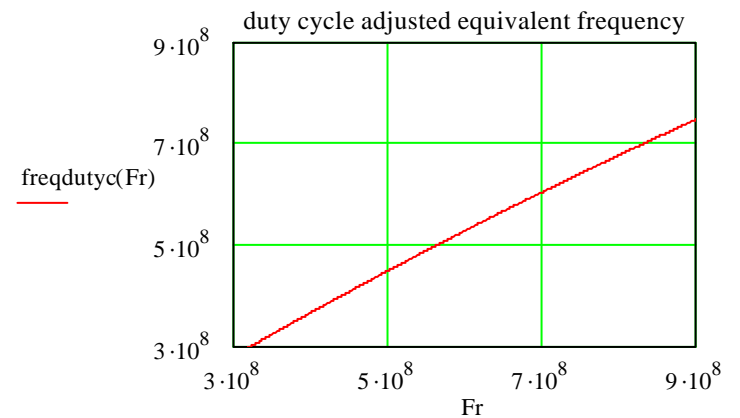
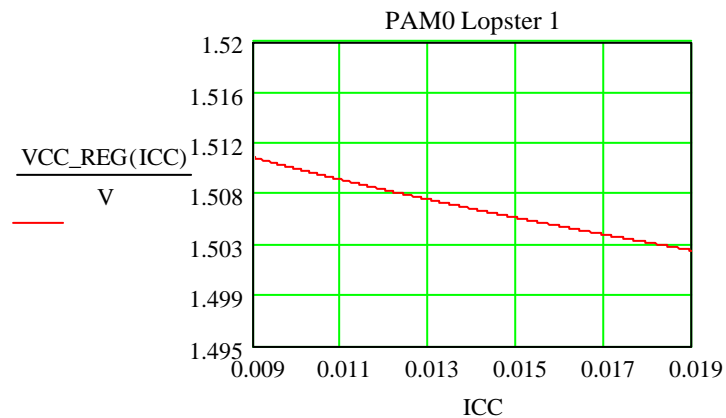
Duty Cycle

$$duty(c(freq)) := 1.001906 + 0.0004526 \left(\frac{freq}{1 \times 10^6 \cdot \text{Hz}} \right) \quad freqduty(c(freq)) := \frac{freq}{\left(\frac{duty(c(freq)) + 1}{2} \right)}$$

The duty cycle is defined as the off/on ratio and this affects the frequency of the series tuned circuit

$$duty(c(freq)) = 1.198$$

$$freqduty(c(freq)) = 394.844 \text{ MHz}$$



Sokal's equations

$$R_{load} := \frac{1}{(34.222 \cdot \text{freqduty}(freq) \cdot C1_eff)} \cdot \left(0.99836 + \frac{0.91394}{Q_L} - \frac{1.0316}{Q_L^2} \right)$$

$$L_2 := \frac{Q_L \cdot R_{load}}{2 \cdot \pi \cdot \text{freqduty}(freq)} - L_{pcb} - L_{lopster}$$

$$C_2 := \frac{1}{(2 \cdot \pi \cdot \text{freqduty}(freq) \cdot R_{load})} \cdot \frac{1}{(Q_L - 0.1048)} \cdot \left[1.0012 + \frac{1.0147}{(Q_L - 1.7979)} \right]$$

$$P_{out_ideal} := \frac{(V_{cc_eff} - V_o)^2}{R_{load}} \cdot 0.576801 \cdot \left(1.0012 - \frac{0.4517}{Q_L} - \frac{0.4024}{Q_L^2} \right)$$

$$P_{out_real} := \frac{(V_{cc_eff} - V_o)^2}{R_{load} + 1.365 \cdot \text{Tran_R} + \text{ESRL2}} \cdot 0.576801 \cdot \left(1.0012 - \frac{0.4517}{Q_L} - \frac{0.4024}{Q_L^2} \right)$$

Rload here is calculated from equation 5 [Sokal]. This equation is modified as the effect of L1 on C1 has already been compensated for above and then re-arranged.

L2 here is the actual inductor value as if it were a physical component and is reduced by Lpcb and Llopster

Account for real losses by taking into account the Transistor on-resistance, Transistor source inductance and the ESR of L2. This is modified to include the impedance of the source inductor. Otherwise this is directly from the reference.

Efficiency

$$\text{Efficiency} := 1 - \left(\frac{\frac{\text{Tran_R_proposed}(V_{Ron})}{1.36}}{R_{load} + \frac{\text{Tran_R_proposed}(V_{Ron})}{1.36}} \right)$$

This is derived from simulations of an ideal amplifier

Efficiency = 0.848

Design Procedure

Step 1.

Enter these values. L_{pcb} is the effective inductance of the pcb from the output pin of Lopster to Rload. C_{pcb} is on the output pin of Lopster to gnd. C_{pcb2} is between L2 & C2

These are the only PCB parasitics considered in the design.

=>

$$freq \equiv 434 \text{ MHz}$$

$$C1 \equiv 1 \text{ pF}$$

$$L_{lopster} \equiv 1.8 \text{ nH}$$

$$L_{pcb} \equiv 5 \text{ nH}$$

$$C_{pcb1} \equiv 0.580 \text{ pF}$$

$$CapPackage \equiv 0.1 \text{ pF}$$

$$C_{pcb2} \equiv 0.240 \text{ pF}$$

$$Tran_L \equiv 0.6 \text{ nH}$$

$$C_{pcb3} \equiv 0.280 \text{ pF}$$

Step 2.

Set VCC

$$VCC_REG(I_{cc}) = 1.509 \text{ V}$$

=>

$$V_{cc} \equiv 1.509 \text{ V}$$

Step 3.

Set the lopster capacitance. Add 100fF for package.

$$CapLopster(V_{off}) + CapPackage = 0.77 \text{ pF}$$

=>

$$C_{lopster} \equiv 0.77 \text{ pF}$$

Step 4.

The on-resistance of the PA transistors and simulations show that this changes for ACON and the voltage across the transistors.

$$I_{cc} = 10.632 \text{ mA}$$

$$V_{Ron} = 0.212 \text{ V}$$

$$Tran_R_{proposed}(V_{Ron}) = 9.961 \text{ ohm}$$

=>

$$Tran_R \equiv 9.961 \text{ ohm}$$

Step 5.

From the suggested L1 select the real L1 with its self resonance and ESR. Use a LQW18 high performance 0603 component series from Murata as these have less loss.

Note: use $SRF = 1.1 \times \text{spec limit SRF}$
use $ESR = 0.9 \times \text{spec limit ESR}$

$$L1_suggest = 202.928 \text{ nH}$$

=>

$$L1 \equiv 180 \text{ nH}$$

$$SRFL1 \equiv 1.3 \cdot 1.1 \text{ GHz}$$

$$ESRL1 \equiv 2.2 \cdot 0.9 \text{ ohm}$$

$$Pout_ideal = 18.207 \text{ mW}$$

Step 6.

Adjust the $Pout_guess$ until the two values converge.

$$Pout_real = 13.6 \text{ mW}$$

=>

$$Pout_guess \equiv 13.605 \text{ mW}$$

Step 7.

Adjust QL to get a good value for C2. QL must be greater than 1.8.

$$Q_L \equiv 3.474$$

=>

$$C_2 = 4.699 \text{ pF}$$

Step 8.

The PCB inductance is already subtracted and from L2 and so get the SRF and ESR assuming LQW15 or LQW18 high performance 0402. (This as component may not actually be used so choose data for the nearest equivalent)

$$L_2 = 50.47 \text{ nH}$$

=>

$$\text{SRFL2} \equiv 2.7 \cdot 1.1 \cdot \text{GHz}$$

$$\text{ESRL2} \equiv 0.29 \cdot 0.9 \cdot \text{ohm}$$

Note: use SRF = 1.1 x spec limit SRF
use ESR = 0.9 x spec limit ESR

Step 9.

From the calculated efficiency put Eff. This has a secondary effect on the RON and the voltage across the transistors.

$$\text{Efficiency} = 0.848$$

=>

$$\text{Eff} \equiv 0.848$$

AT THIS POINT REPEAT FROM STEP 1 UNTIL CONVERGENCE

Step 10.

These values affect the added LPF. FR is the ratio of the filter cutoff frequency to the output frequency and should be selected to have the minimum loss at the required frequency. Lar_db is the filter ripple. A higher ripple reduces the harmonics, but also increases the losses.

$$\text{FR} \equiv 1.17$$

$$L_{\text{ar_db}} \equiv 2.4$$

Step 11

The notch_ratio defines the notch, too low and the notch is too wide, too high and the impedance at the wanted frequency changes too much. Suggest 1.2 to 1.6. Default 1.40

$$\text{notch_ratio} \equiv 1.4$$

Step 12.

If Rload is < 50 Ohms then goto matching 1 to 4.
If Rload is > 50 Ohms then goto matching 5 to 8

$$R_{\text{load}} = 40.9 \text{ ohm}$$

Design the Impedance Match to 50 Ohms, when Rload > 50 Ohms

These equations are for Rload greater than 50 ohms, with the match following the series LC. The starting component is a capacitor to gnd at Rload, followed by a series inductor at 50 ohms.

$$Q_{\text{match}} := \sqrt{\frac{R_{\text{load}}}{50 \cdot \text{ohm}} - 1}$$

$$C_{\text{gnd}_1} := \frac{Q_{\text{match}}}{2 \cdot \pi \cdot \text{freq} \cdot R_{\text{load}}}$$

$$L_{\text{series}_1} := \frac{Q_{\text{match}} \cdot 50 \cdot \text{ohm}}{2 \cdot \pi \cdot \text{freq}}$$

$$Q_{\text{match}} = 0.427i$$

$$C_{\text{gnd}_1} = 3.825i \text{ pF}$$

$$L_{\text{series}_1} = 7.822i \text{ nH}$$

Design the Impedance Match to 50 Ohms, when Rload < 50 Ohms

These equations are for Rload less than 50 ohms.

The series inductor at Rload will be added to the resonant L, the capacitor to gnd is at 50 ohms

$$Q_{\text{match}} := \sqrt{\frac{50 \cdot \text{ohm}}{R_{\text{load}}} - 1}$$

$$L_{\text{series}_2} := \frac{Q_{\text{match}} \cdot R_{\text{load}}}{2 \cdot \pi \cdot \text{freq}}$$

$$C_{\text{gnd}_2} := \frac{Q_{\text{match}}}{2 \cdot \pi \cdot \text{freq} \cdot 50 \cdot \text{ohm}}$$

$$L_{\text{series_modified}_2} := L_2 + L_{\text{series}_2}$$

$$Q_{\text{match}} = 0.472$$

$$L_{\text{series}_2} = 7.075 \text{ nH}$$

$$C_{\text{gnd}_2} = 3.46 \text{ pF}$$

Note both the IC and the pcb has inductance which has already been subtracted.

Add a further PI section filter if required

This is to add a further LC pole to remove harmonics. This value is the cutoff ratio **FR * freq** and is set for 15%. This is the ratio of the third order Tchebychev peak to the Fc of the filter.

When Rload > 50 ohm, a three stage filter is proposed starting with series L.

$$L_{g1} := \frac{50 \cdot \text{ohm} \cdot g_1}{2 \cdot \pi \cdot \text{FR} \cdot \text{freq}}$$

$$C_{g2} := \frac{g_2}{50 \cdot \text{ohm} \cdot 2 \cdot \pi \cdot \text{FR} \cdot \text{freq}}$$

$$L_{g3} := \frac{50 \cdot \text{ohm} \cdot g_3}{2 \cdot \pi \cdot \text{FR} \cdot \text{freq}}$$

$$L_{\text{series_modified}_1} := L_{\text{series}_1} + L_{g1}$$

$$L_{\text{series_modified}_1} = 46.502 + 7.822i \text{ nH}$$

$$C_{g2} = 4.893 \text{ pF}$$

$$L_{g3} = 46.502 \text{ nH}$$

when Rload < 50 ohm, a three stage filter is proposed starting with cap to ground.

$$C_{g1} := \frac{g_1}{50 \cdot \text{ohm} \cdot 2 \cdot \pi \cdot \text{FR} \cdot \text{freq}}$$

$$L_{g2} := \frac{50 \cdot \text{ohm} \cdot g_2}{2 \cdot \pi \cdot \text{FR} \cdot \text{freq}}$$

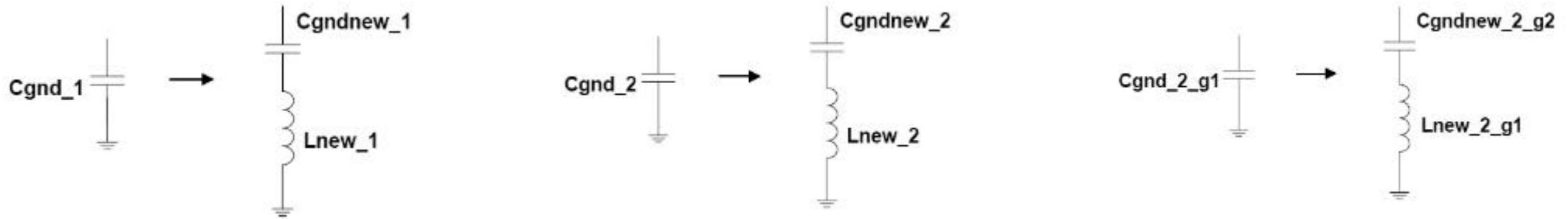
$$C_{g3} := \frac{g_3}{50 \cdot \text{ohm} \cdot 2 \cdot \pi \cdot \text{FR} \cdot \text{freq}}$$

$$C_{g1} = 18.601 \text{ pF}$$

$$L_{g2} = 12.232 \text{ nH}$$

$$C_{g3} = 18.601 \text{ pF}$$

Make the first capacitor into a notch for the three possible designs.



The capacitor to ground can be replaced by a notch to target the second harmonic which is usually quite high in a class-E amplifier. The Q of the notch is set by the notch_ratio, a low number is very selective and component tolerances may mistune this to have little effect. A high number is a broader notch with that will give less-peak attenuation. The ratio of 1.4 is about right for most applications.

Convert the capacitor to ground (Cgnd) to a series LC notch to attenuate the second harmonic, when Rload > 50 Ohms

$$C_{gndnew_1} := \frac{C_{gnd_1}}{\text{notch_ratio}} \quad Z_{now_1} := \frac{1}{2 \cdot \pi \cdot \text{freq} \cdot C_{gnd_1}} \quad L_{new_1} := \frac{1}{(4 \cdot \pi \cdot \text{freq})^2 \cdot C_{gndnew_1}} \quad Z_{new_1} := \frac{1}{2 \cdot \pi \cdot \text{freq} \cdot C_{gndnew_1}} - 2 \cdot \pi \cdot \text{freq} \cdot L_{new_1}$$

$$C_{gnd_1} = 3.825 \text{ i pF} \Rightarrow \begin{matrix} C_{gndnew_1} = 2.732 \text{ i pF} & Z_{now_1} = -95.872 \text{ i ohm} \\ L_{new_1} = -12.305 \text{ i nH} & Z_{new_1} = -100.665 \text{ i ohm} \end{matrix}$$

Convert the capacitor to ground (C_{gnd}) to a series LC notch to attenuate the second harmonic, when R_{load} < 50 Ohms

$$C_{\text{gndnew}_2} := \frac{C_{\text{gnd}_2}}{\text{notch_ratio}} \quad Z_{\text{now}_2} := \frac{1}{2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gnd}_2}} \quad L_{\text{new}_2} := \frac{1}{(4 \cdot \pi \cdot \text{freq})^2 \cdot C_{\text{gndnew}_2}} \quad Z_{\text{new}_2} := \frac{1}{2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gndnew}_2}} - 2 \cdot \pi \cdot \text{freq} \cdot L_{\text{new}_2}$$

$$C_{\text{gnd}_2} = 3.46 \text{ pF} \quad \Rightarrow \quad \begin{array}{l} C_{\text{gndnew}_2} = 2.471 \text{ pF} \\ L_{\text{new}_2} = 13.605 \text{ nH} \end{array} \quad \begin{array}{l} Z_{\text{now}_2} = 106.002 \text{ ohm} \\ Z_{\text{new}_2} = 111.302 \text{ ohm} \end{array}$$

Convert the capacitor to ground (C_{gnd}+C_{g1}) to a series LC notch to attenuate the second harmonic, when R_{load} < 50 Ohms.

$$C_{\text{gnd}_2_g1} := C_{\text{gnd}_2} + C_{g1} \quad C_{\text{gndnew}_2_g1} := \frac{C_{\text{gnd}_2_g1}}{\text{notch_ratio}} \quad Z_{\text{now}_3} := \frac{1}{2 \cdot \pi \cdot \text{freq} \cdot (C_{\text{gnd}_2} + C_{g1})} \quad L_{\text{new}_2_g1} := \frac{1}{(4 \cdot \pi \cdot \text{freq})^2 \cdot C_{\text{gndnew}_2_g1}}$$

$$Z_{\text{new}_3} := \frac{1}{2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gndnew}_2_g1}} - 2 \cdot \pi \cdot \text{freq} \cdot L_{\text{new}_2_g1} \quad C_{\text{gnd}_2_g1} = 22.06 \text{ pF} \quad \Rightarrow \quad \begin{array}{l} C_{\text{gndnew}_2_g1} = 15.757 \text{ pF} \\ L_{\text{new}_2_g1} = 2.134 \text{ nH} \end{array} \quad \begin{array}{l} Z_{\text{now}_3} = 16.623 \text{ ohm} \\ Z_{\text{new}_3} = 17.454 \text{ ohm} \end{array}$$

Calculate Harmonics out of the class-E tuned network

These harmonics (-dBc) are out of the series LC tuned network. They are calculated according to N O Sokal, F H Raab, Harmonic output of class E PA and load coupling network design JSSC Feb 1977. These equations are modified according to N O Sokal "Improved Design Equations", last paragraph.

$$I_n/I_1 \approx (c_n/c_1) (Z_1/Z_n) \quad (1)$$

$$Z_1/Z_n \approx \left(\frac{1.42}{nQ_L}\right) \left[\frac{1}{\left(1 - 1/n^2\right) - (0.66 - 2.08/n^2)/Q_L} \right] \quad (2)$$

in [1]-[6], when the switch duty ratio D is 50%. (b) In (2), change the factor 1.42 to 1.0147; the factor 2.08 to 1.7879; and the factor 0.66 to 0.773. (c) Recalculate the numerical

n	c _n	c _n /c ₁
1	1.639	1.000
2	0.8477	0.5172
3	0.2222	0.1356
4	0.1432	0.08737
5	0.07997	0.04879
6	0.05907	0.03604
7	0.04079	0.02489
8	0.03235	0.01974
9	0.02467	0.01505
10	0.02045	0.01248

$$H2 := 20 \cdot \log \left[0.5172 \cdot \frac{1.0147}{2 \cdot Q_L} \cdot \left[\frac{1}{\left(1 - \frac{1}{2^2}\right) - \frac{\left(0.773 - \frac{1.7879}{2^2}\right)}{Q_L}} \right] \right]$$

$$H3 := 20 \cdot \log \left[0.1356 \cdot \frac{1.0147}{3 \cdot Q_L} \cdot \left[\frac{1}{\left(1 - \frac{1}{3^2}\right) - \frac{\left(0.773 - \frac{1.7879}{3^2}\right)}{Q_L}} \right] \right]$$

$$H4 := 20 \cdot \log \left[0.08737 \cdot \frac{1.0147}{4 \cdot Q_L} \cdot \left[\frac{1}{\left(1 - \frac{1}{4^2}\right) - \frac{\left(0.773 - \frac{1.7879}{4^2}\right)}{Q_L}} \right] \right]$$

$$H5 := 20 \cdot \log \left[0.04879 \cdot \frac{1.0147}{5 \cdot Q_L} \cdot \left[\frac{1}{\left(1 - \frac{1}{5^2}\right) - \frac{\left(0.773 - \frac{1.7879}{5^2}\right)}{Q_L}} \right] \right]$$

$$H6 := 20 \cdot \log \left[0.03604 \cdot \frac{1.0147}{6 \cdot Q_L} \cdot \left[\frac{1}{\left(1 - \frac{1}{6^2}\right) - \frac{\left(0.773 - \frac{1.7879}{6^2}\right)}{Q_L}} \right] \right]$$

$$H7 := 20 \cdot \log \left[0.02489 \cdot \frac{1.0147}{7 \cdot Q_L} \cdot \left[\frac{1}{\left(1 - \frac{1}{7^2}\right) - \frac{\left(0.773 - \frac{1.7879}{7^2}\right)}{Q_L}} \right] \right]$$

$$H2 = -18.777$$

$$H5 = -48.497$$

$$H3 = -34.777$$

$$H6 = -52.779$$

$$H4 = -41.372$$

$$H7 = -57.374$$

REFERENCE - ideal frequency response of the series tuned circuit

The series tuned circuit has a frequency response that we need to measure to compensate for the losses of this circuit in the networks below. The reason for this complexity is that the harmonics are calculated relying on a wideband constant R_{load} , but the extra filtering and impedance conversion on the output of the tuned circuit presents R_{load} only at the output frequency and not at the harmonic frequencies.

$$L_2 = 50.47 \text{ nH}$$

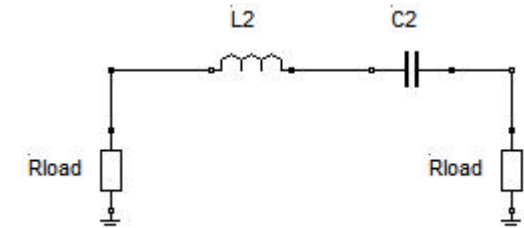
$$C_2 = 4.699 \text{ pF}$$

$$R_{load} = 40.9 \text{ ohm}$$

$$ESRL_2 = 0.261 \text{ ohm}$$

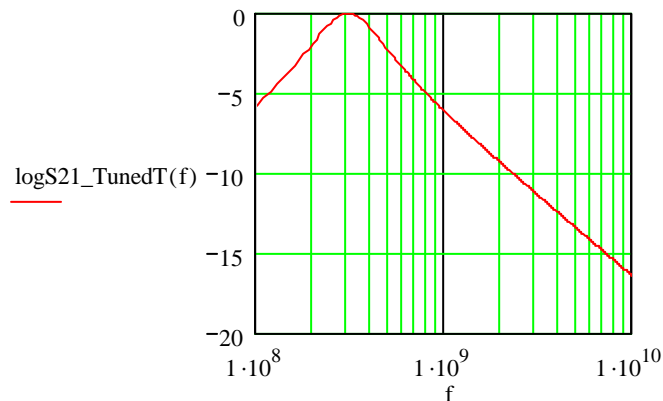
$$\text{TunedL}(\text{freq}) := \begin{bmatrix} 1 & \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot (L_2 + L_{pcb} + L_{lopster})}{\text{ohm}} + \frac{ESRL_2}{\text{ohm}} \\ 0 & 1 \end{bmatrix}$$

$$\text{TunedC}(\text{freq}) := \begin{bmatrix} 1 & \frac{1}{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_2 \cdot \text{ohm}} \\ 0 & 1 \end{bmatrix}$$



$$\text{TunedT}(\text{freq}) := \text{TunedL}(\text{freq}) \cdot \text{TunedC}(\text{freq})$$

$$\log S_{21_TunedT}(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{load}^2}{\text{ohm}^2} \right)^{0.5}}{\text{TunedT}(\text{freq})_{0,0} \cdot \frac{R_{load}}{\text{ohm}} + \text{TunedT}(\text{freq})_{0,1} + \frac{R_{load}^2}{\text{ohm}^2} \cdot \text{TunedT}(\text{freq})_{1,0} + \text{TunedT}(\text{freq})_{1,1} \cdot \frac{R_{load}}{\text{ohm}}} \right], 10$$



$$T1dB := \log S_{21_TunedT}(\text{freq})$$

$$T1dB = -1.415$$

$$T2dB := \log S_{21_TunedT}(2 \cdot \text{freq})$$

$$T2dB = -5.427$$

$$T3dB := \log S_{21_TunedT}(3 \cdot \text{freq})$$

$$T3dB = -7.405$$

$$T4dB := \log S_{21_TunedT}(4 \cdot \text{freq})$$

$$T4dB = -8.731$$

$$T5dB := \log S_{21_TunedT}(5 \cdot \text{freq})$$

$$T5dB = -9.735$$

$$T6dB := \log S_{21_TunedT}(6 \cdot \text{freq})$$

$$T6dB = -10.547$$

$$T7dB := \log S_{21_TunedT}(7 \cdot \text{freq})$$

$$T7dB = -11.228$$

Now modify the harmonics with the matching network. There four matching networks are for Rload < 50 Ohms

This section subtracts from the harmonics the extra attenuation obtained from the impedance matching network. Four networks have been designed for Rload<50R, and four networks for >50R load, giving eight in total. Half of these have a second harmonic notch. Half of these use a second LC filter stage. This calculation is done using the ABCD matrixies which are multiplied together and converted to S21 using standard equations. The frequency response of all networks are shown below, the notch helps with the second harmonic but higher harmonics get through more easily. The more complexity the more loss and as a simple approximation for loss, all the inductors have 1ohm in series in the equations.

1

$$L_{\text{series_2}} = 7.075 \text{ nH}$$

$$C_{\text{gnd_2}} = 3.46 \text{ pF}$$

$$A(\text{freq}) := \begin{pmatrix} 1 & \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{\text{series_2}}}{\text{ohm}} + \frac{\text{ESRL2}}{\text{ohm}} \\ 0 & 1 \end{pmatrix}$$

$$B(\text{freq}) := \begin{pmatrix} 1 & 0 \\ j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gnd_2}} \cdot \text{ohm} & 1 \end{pmatrix}$$

$$\log S21_1(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{\text{load}} \cdot 50}{\text{ohm}} \right)^{0.5}}{D(\text{freq})_{0,0} \cdot 50 + D(\text{freq})_{0,1} + \frac{50 \cdot R_{\text{load}}}{\text{ohm}} \cdot D(\text{freq})_{1,0} + D(\text{freq})_{1,1} \cdot \frac{R_{\text{load}}}{\text{ohm}}} \right], 10$$

$$\log S21_1(\text{freq}) = -1.797$$

$$T1\text{dB} = -1.415$$

$$10 \cdot \log \left(\frac{\text{Pout_real}}{\text{mW}} \right) = 11.335$$

2

$$L_{\text{series_2}} = 7.075 \text{ nH}$$

$$C_{\text{gndnew_2}} = 2.471 \text{ pF}$$

$$L_{\text{new_2}} = 13.605 \text{ nH}$$

$$C(\text{freq}) := \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ \frac{1}{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gndnew_2}} \cdot \text{ohm}} + \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{\text{new_2}}}{\text{ohm}} + 1 & 1 \end{pmatrix}$$

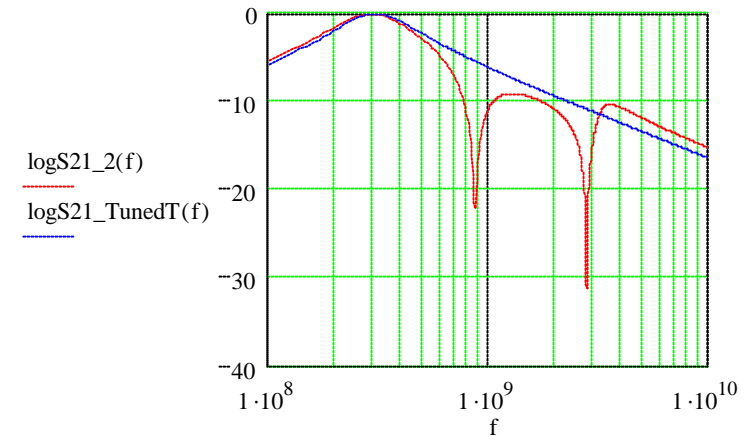
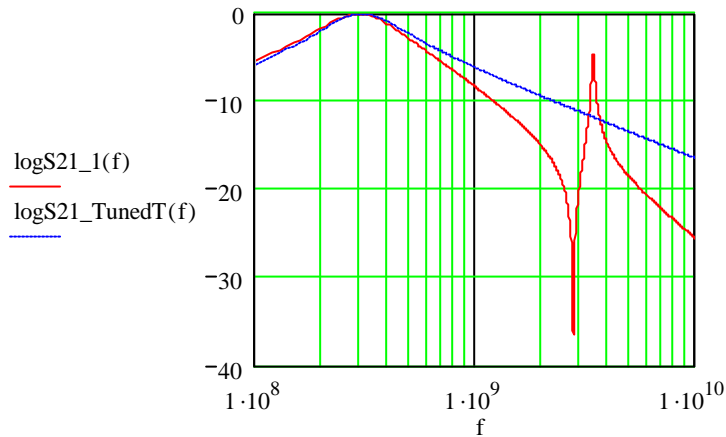
$$D(\text{freq}) := \text{TunedTR}(\text{freq}) \cdot A(\text{freq}) \cdot B(\text{freq})$$

$$E(\text{freq}) := \text{TunedTR}(\text{freq}) \cdot A(\text{freq}) \cdot C(\text{freq})$$

$$\log S21_2(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{\text{load}} \cdot 50}{\text{ohm}} \right)^{0.5}}{E(\text{freq})_{0,0} \cdot 50 + E(\text{freq})_{0,1} + \frac{50 \cdot R_{\text{load}}}{\text{ohm}} \cdot E(\text{freq})_{1,0} + E(\text{freq})_{1,1} \cdot \frac{R_{\text{load}}}{\text{ohm}}} \right], 10$$

$$Pout_real_1_dBm := 10 \cdot \log\left(\frac{Pout_real}{mW}\right) + \log S_{21_1}(freq) - T1dB$$

$$Pout_real_2_dBm := 10 \cdot \log\left(\frac{Pout_real}{mW}\right) + \log S_{21_2}(freq) - T1dB$$



Pout_real_1_dBm = 10.954

Pout_real_2_dBm = 10.948

$$H2_1 := H2 + \log S_{21_1}(2freq) - T2dB$$

$$H2_1 = -20.518$$

$$H2_2 := H2 + \log S_{21_2}(2freq) - T2dB$$

$$H2_2 = -36.282$$

$$H3_1 := H3 + \log S_{21_1}(3freq) - T3dB$$

$$H3_1 = -37.951$$

$$H3_2 := H3 + \log S_{21_2}(3freq) - T3dB$$

$$H3_2 = -36.467$$

$$H4_1 := H4 + \log S_{21_1}(4freq) - T4dB$$

$$H4_1 = -46.013$$

$$H4_2 := H4 + \log S_{21_2}(4freq) - T4dB$$

$$H4_2 = -42.64$$

$$H5_1 := H5 + \log S_{21_1}(5freq) - T5dB$$

$$H5_1 = -54.971$$

$$H5_2 := H5 + \log S_{21_2}(5freq) - T5dB$$

$$H5_2 = -50.447$$

$$H6_1 := H6 + \log S_{21_1}(6freq) - T6dB$$

$$H6_1 = -63.376$$

$$H6_2 := H6 + \log S_{21_2}(6freq) - T6dB$$

$$H6_2 = -58.105$$

$$H7_1 := H7 + \log S_{21_1}(7freq) - T7dB$$

$$H7_1 = -64.049$$

$$H7_2 := H7 + \log S_{21_2}(7freq) - T7dB$$

$$H7_2 = -59.636$$

$$L1 = 180 \text{ nH}$$

$$C1 = 1 \text{ pF}$$

$$Icc = 10.632 \text{ mA}$$

$$L1 = 180 \text{ nH}$$

$$C1 = 1 \text{ pF}$$

$$Icc = 10.632 \text{ mA}$$

$$C2 = 4.699 \text{ pF}$$

$$C2 = 4.699 \text{ pF}$$

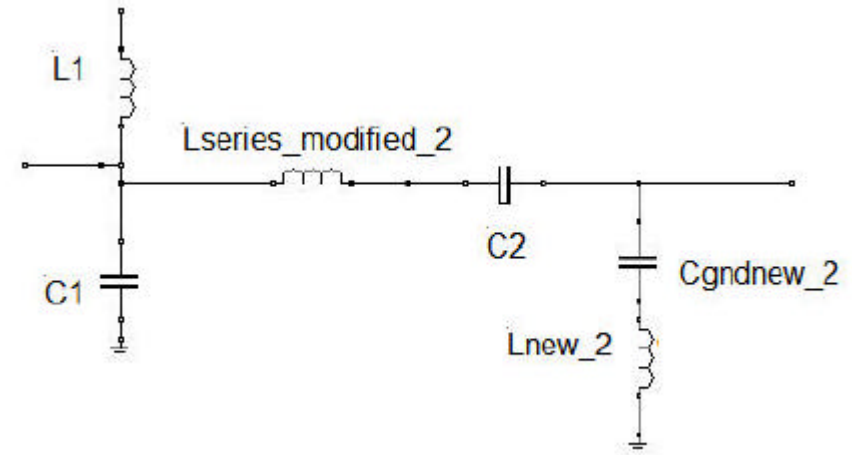
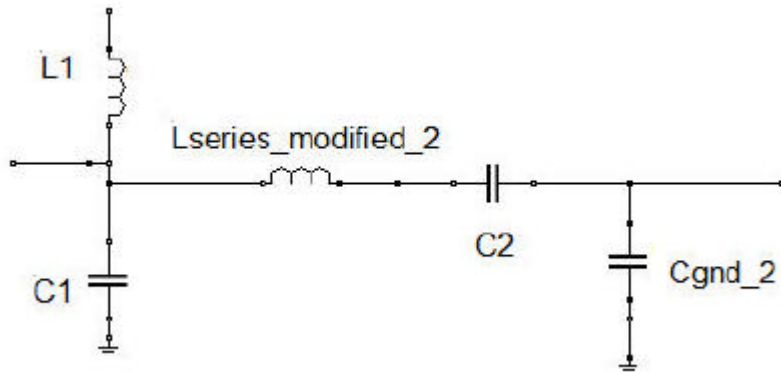
$$C_{gndnew_2} = 2.471 \text{ pF}$$

$$L_{series_modified_2} = 57.548 \text{ nH}$$

$$C_{gnd_2} = 3.46 \text{ pF}$$

$$L_{series_modified_2} = 57.548 \text{ nH}$$

$$L_{new_2} = 13.605 \text{ nH}$$



3

$$L_{\text{series}_2} = 7.075 \text{ nH}$$

$$L_{g2} = 12.232 \text{ nH}$$

$$C_{\text{gnd}_2_g1} = 22.06 \text{ pF}$$

$$C_{g3} = 18.601 \text{ pF}$$

$$F(\text{freq}) := \begin{pmatrix} 1 & 0 \\ j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gnd}_2_g1} \cdot \text{ohm} & 1 \end{pmatrix}$$

$$G(\text{freq}) := \begin{pmatrix} 1 & \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{g2}}{\text{ohm}} + 1 \\ 0 & 1 \end{pmatrix}$$

$$H(\text{freq}) := \begin{pmatrix} 1 & 0 \\ j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gnd}_2_g1} \cdot \text{ohm} & 1 \end{pmatrix}$$

4

$$L_{\text{series}_2} = 7.075 \text{ nH}$$

$$L_{g2} = 12.232 \text{ nH}$$

$$C_{\text{gndnew}_2_g1} = 15.757 \text{ pF}$$

$$C_{g3} = 18.601 \text{ pF}$$

$$L_{\text{new}_2_g1} = 2.134 \text{ nH}$$

$$J(\text{freq}) := \begin{pmatrix} 1 & 0 \\ \frac{1}{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gndnew}_2_g1} \cdot \text{ohm} + \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{\text{new}_2_g1}}{\text{ohm}} + 1} & 1 \end{pmatrix}$$

$$I(\text{freq}) := \text{TunedTR}(\text{freq}) \cdot A(\text{freq}) \cdot F(\text{freq}) \cdot G(\text{freq}) \cdot H(\text{freq})$$

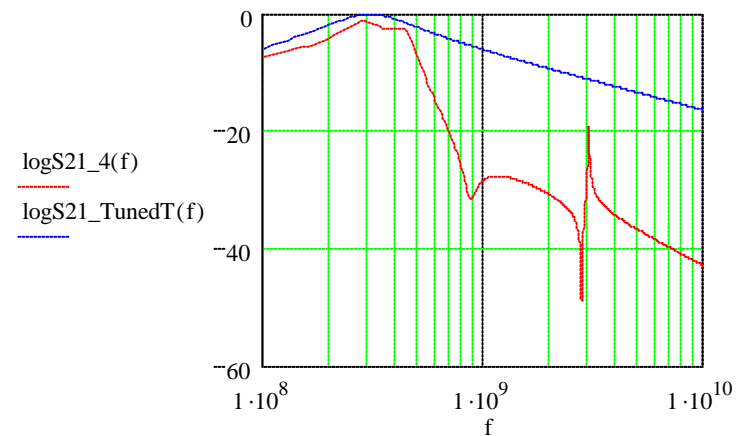
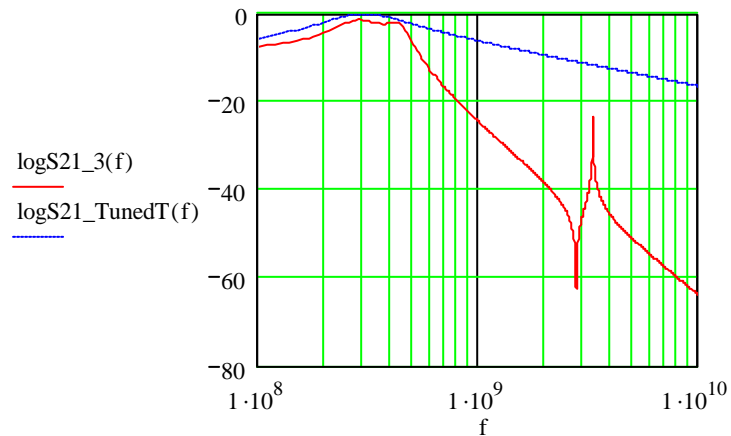
$$K(\text{freq}) := \text{TunedTR}(\text{freq}) \cdot A(\text{freq}) \cdot J(\text{freq}) \cdot G(\text{freq}) \cdot H(\text{freq})$$

$$\log S_{21_3}(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{\text{load}} \cdot 50}{\text{ohm}} \right)^{0.5}}{I(\text{freq})_{0,0} \cdot 50 + I(\text{freq})_{0,1} + \frac{50 \cdot R_{\text{load}}}{\text{ohm}} \cdot I(\text{freq})_{1,0} + I(\text{freq})_{1,1} \cdot \frac{R_{\text{load}}}{\text{ohm}}} \right], 10$$

$$\log S_{21_4}(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{\text{load}} \cdot 50}{\text{ohm}} \right)^{0.5}}{K(\text{freq})_{0,0} \cdot 50 + K(\text{freq})_{0,1} + \frac{50 \cdot R_{\text{load}}}{\text{ohm}} \cdot K(\text{freq})_{1,0} + K(\text{freq})_{1,1} \cdot \frac{R_{\text{load}}}{\text{ohm}}} \right], 10$$

$$\text{Pout_real_3_dBm} := 10 \cdot \log \left(\frac{\text{Pout_real}}{\text{mW}} \right) + \log S_{21_3}(\text{freq}) - \text{T1dB}$$

$$\text{Pout_real_4_dBm} := 10 \cdot \log \left(\frac{\text{Pout_real}}{\text{mW}} \right) + \log S_{21_4}(\text{freq}) - \text{T1dB}$$



Pout_real_3_dBm = 10.824

$H2_3 := H2 + \log S21_3(2\text{freq}) - T2\text{dB}$

$H2_3 = -34.839$

$H3_3 := H3 + \log S21_3(3\text{freq}) - T3\text{dB}$

$H3_3 = -57.111$

$H4_3 := H4 + \log S21_3(4\text{freq}) - T4\text{dB}$

$H4_3 = -68.101$

$H5_3 := H5 + \log S21_3(5\text{freq}) - T5\text{dB}$

$H5_3 = -79.128$

$H6_3 := H6 + \log S21_3(6\text{freq}) - T6\text{dB}$

$H6_3 = -89.028$

$H7_3 := H7 + \log S21_3(7\text{freq}) - T7\text{dB}$

$H7_3 = -90.33$

$L1 = 180\text{nH}$ $C1 = 1\text{pF}$

$I_{cc} = 10.632\text{mA}$

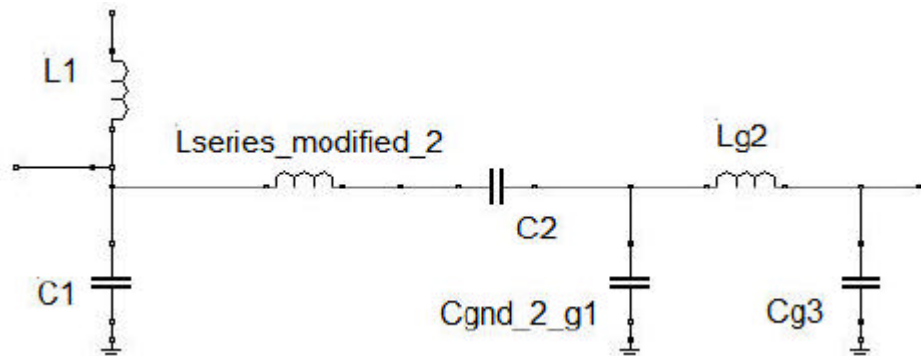
$C2 = 4.699\text{pF}$

$L_{\text{series_modified_2}} = 57.548\text{nH}$

$C_{\text{gnd_2_g1}} = 22.06\text{pF}$

$L_{g2} = 12.232\text{nH}$

$C_{g3} = 18.601\text{pF}$



Pout_real_4_dBm = 10.237

$H2_4 := H2 + \log S21_4(2\text{freq}) - T2\text{dB}$

$H2_4 = -44.746$

$H3_4 := H3 + \log S21_4(3\text{freq}) - T3\text{dB}$

$H3_4 = -55.272$

$H4_4 := H4 + \log S21_4(4\text{freq}) - T4\text{dB}$

$H4_4 = -62.278$

$H5_4 := H5 + \log S21_4(5\text{freq}) - T5\text{dB}$

$H5_4 = -70.467$

$H6_4 := H6 + \log S21_4(6\text{freq}) - T6\text{dB}$

$H6_4 = -77.411$

$H7_4 := H7 + \log S21_4(7\text{freq}) - T7\text{dB}$

$H7_4 = -70.624$

$L1 = 180\text{nH}$ $C1 = 1\text{pF}$

$I_{cc} = 10.632\text{mA}$

$C2 = 4.699\text{pF}$

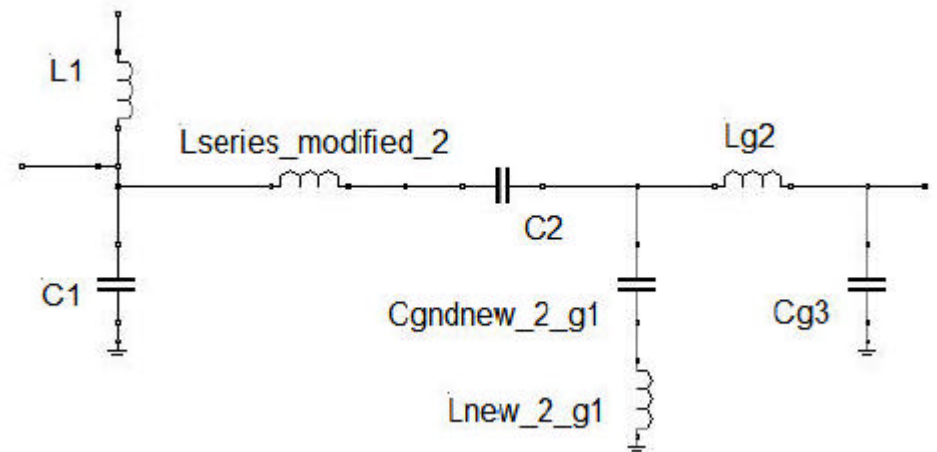
$L_{\text{series_modified_2}} = 57.548\text{nH}$

$C_{\text{gndnew_2_g1}} = 15.757\text{pF}$

$L_{g2} = 12.232\text{nH}$

$L_{\text{new_2_g1}} = 2.134\text{nH}$

$C_{g3} = 18.601\text{pF}$



These four matching networks are for $R_{load} > 50 \text{ Ohms}$

5

$$C_{gnd_1} = 3.825 \text{ pF}$$

$$L_{series_1} = 7.822 \text{ nH}$$

$$L(\text{freq}) := \begin{pmatrix} 1 & 0 \\ j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{gnd_1} \cdot \text{ohm} & 1 \end{pmatrix} \quad M(\text{freq}) := \begin{pmatrix} 1 & \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{series_1}}{\text{ohm}} + 1 \\ 0 & 1 \end{pmatrix}$$

6

$$C_{gndnew_1} = 2.732 \text{ pF}$$

$$L_{series_1} = 7.822 \text{ nH}$$

$$L_{new_1} = -12.305 \text{ nH}$$

$$O(\text{freq}) := \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ \frac{1}{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{gndnew_1} \cdot \text{ohm}} + \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{new_1}}{\text{ohm}} + 1 & 1 \end{pmatrix}$$

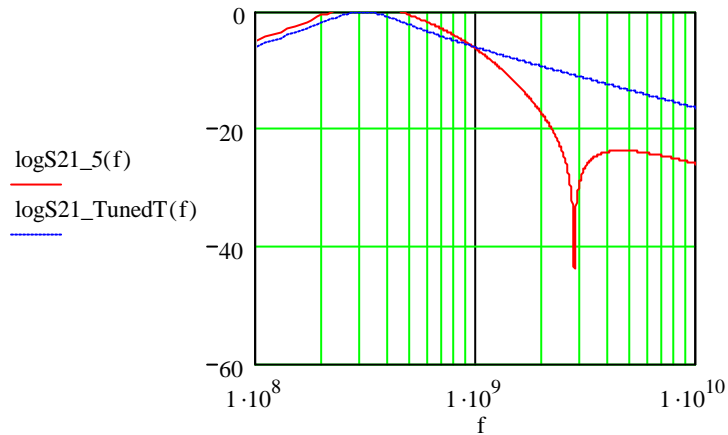
$$N(\text{freq}) := \text{TunedTR}(\text{freq}) \cdot L(\text{freq}) \cdot M(\text{freq})$$

$$P(\text{freq}) := \text{TunedTR}(\text{freq}) \cdot O(\text{freq}) \cdot M(\text{freq})$$

$$\log S_{21_5}(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{load} \cdot 50}{\text{ohm}} \right)^{0.5}}{N(\text{freq})_{0,0} \cdot 50 + N(\text{freq})_{0,1} + \frac{50 \cdot R_{load}}{\text{ohm}} \cdot N(\text{freq})_{1,0} + N(\text{freq})_{1,1} \cdot \frac{R_{load}}{\text{ohm}}}, 10 \right]$$

$$\log S_{21_6}(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{load} \cdot 50}{\text{ohm}} \right)^{0.5}}{P(\text{freq})_{0,0} \cdot 50 + P(\text{freq})_{0,1} + \frac{50 \cdot R_{load}}{\text{ohm}} \cdot P(\text{freq})_{1,0} + P(\text{freq})_{1,1} \cdot \frac{R_{load}}{\text{ohm}}}, 10 \right]$$

$$\text{Pout_real_5_dBm} := 10 \cdot \log\left(\frac{\text{Pout_real}}{\text{mW}}\right) + \log\text{S21_5}(\text{freq}) - \text{T1dB}$$



Pout_real_5_dBm = 12.961

- H2_5 := H2 + logS21_5(3freq) - T2dB
- H3_5 := H3 + logS21_5(3freq) - T3dB
- H4_5 := H4 + logS21_5(4freq) - T4dB
- H5_5 := H5 + logS21_5(5freq) - T5dB
- H6_5 := H6 + logS21_5(6freq) - T6dB
- H7_5 := H7 + logS21_5(7freq) - T7dB

L1 = 180 nH C1 = 1 pF

C2 = 4.699 pF

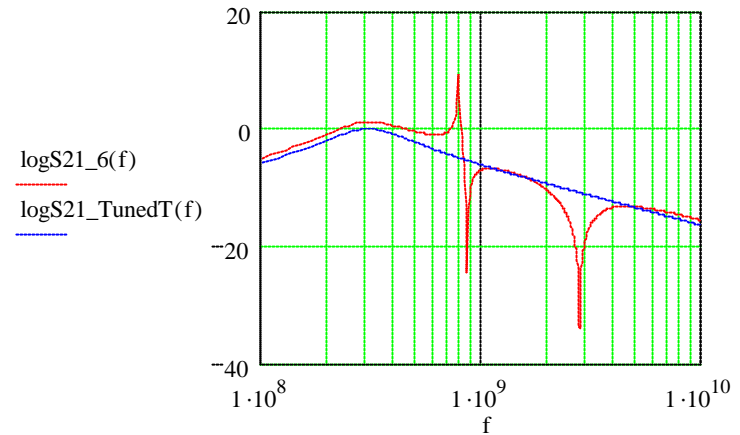
L2 = 50.473 nH

Cgnd_1 = 3.825 i pF

Icc = 10.632 mA

Lseries_1 = 7.822 i nH

$$\text{Pout_real_6_dBm} := 10 \cdot \log\left(\frac{\text{Pout_real}}{\text{mW}}\right) + \log\text{S21_6}(\text{freq}) - \text{T1dB}$$



Pout_real_6_dBm = 12.903

- H2_6 := H2 + logS21_6(2freq) - T2dB
- H3_6 := H3 + logS21_6(3freq) - T3dB
- H4_6 := H4 + logS21_6(4freq) - T4dB
- H5_6 := H5 + logS21_6(5freq) - T5dB
- H6_6 := H6 + logS21_6(6freq) - T6dB
- H7_6 := H7 + logS21_6(7freq) - T7dB

L1 = 180 nH C1 = 1 pF

C2 = 4.699 pF

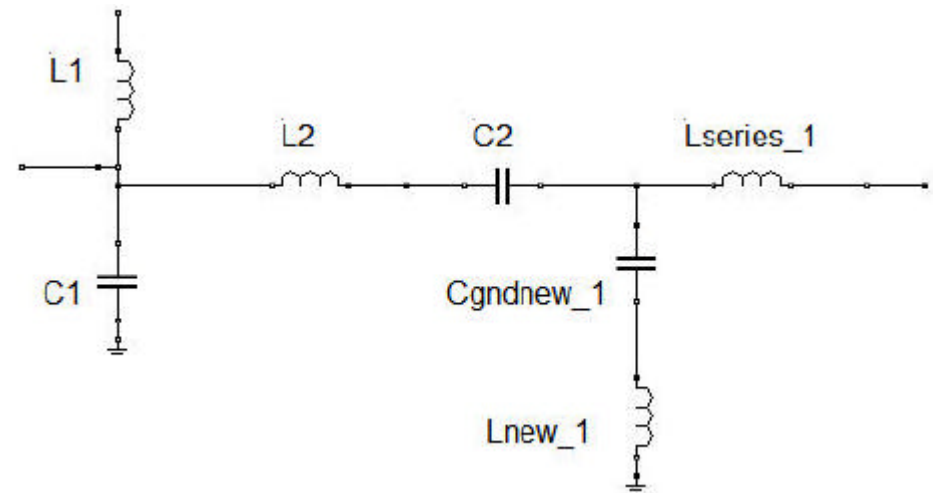
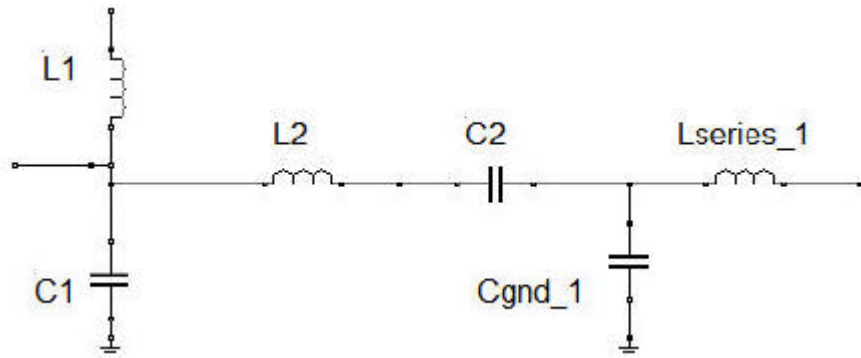
L2 = 50.473 nH

Lseries_1 = 7.822 i nH

Cgndnew_1 = 2.732 i pF

Icc = 10.632 mA

Lnew_1 = -12.305 i nH



7

$$L_{\text{series_modified_1}} = 46.502 + 7.822i \text{ nH} \quad L_{g3} = 46.502 \text{ nH}$$

$$C_{\text{gnd_1}} = 3.825i \text{ pF}$$

$$C_{g2} = 4.893 \text{ pF}$$

$$L(\text{freq}) := \begin{pmatrix} 1 & 0 \\ j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gnd_1}} \cdot \text{ohm} & 1 \end{pmatrix}$$

$$Q(\text{freq}) := \begin{pmatrix} 1 & \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{\text{series_modified_1}}}{\text{ohm}} + 1 \\ 0 & 1 \end{pmatrix}$$

$$R(\text{freq}) := \begin{pmatrix} 1 & 0 \\ j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{g2} \cdot \text{ohm} & 1 \end{pmatrix}$$

$$S(\text{freq}) := \begin{pmatrix} 1 & \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{g3}}{\text{ohm}} + 1 \\ 0 & 1 \end{pmatrix}$$

8

$$L_{\text{series_modified_1}} = 46.502 + 7.822i \text{ nH} \quad L_{g3} = 46.502 \text{ nH}$$

$$C_{\text{gndnew_1}} = 2.732i \text{ pF}$$

$$C_{g2} = 4.893 \text{ pF}$$

$$L_{\text{new_1}} = -12.305i \text{ nH}$$

$$T(\text{freq}) := \begin{pmatrix} 1 & 0 \\ \frac{1}{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot C_{\text{gndnew_1}} \cdot \text{ohm} + \frac{j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L_{\text{new_1}}}{\text{ohm}} + 1} & 1 \end{pmatrix}$$

$$U(\text{freq}) := \text{TunedTR}(\text{freq}) \cdot L(\text{freq}) \cdot Q(\text{freq}) \cdot R(\text{freq}) \cdot S(\text{freq})$$

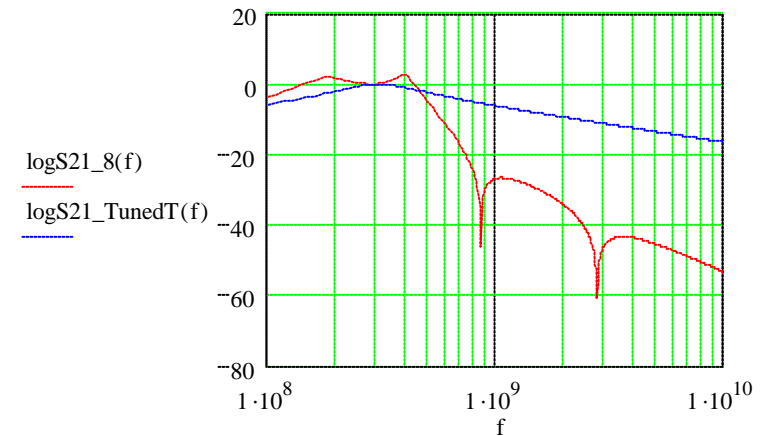
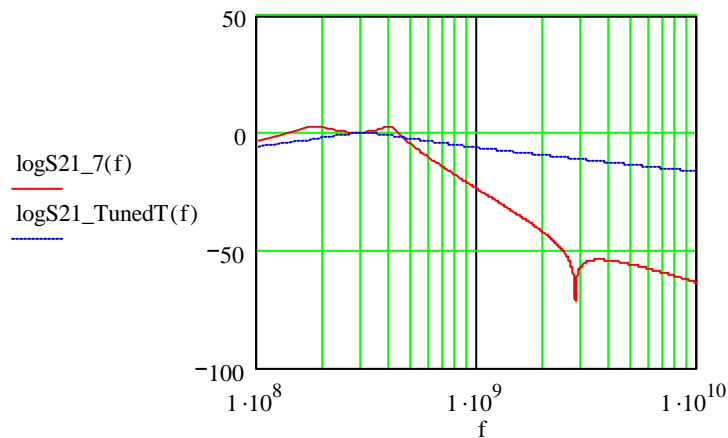
$$V(\text{freq}) := \text{TunedTR}(\text{freq}) \cdot T(\text{freq}) \cdot Q(\text{freq}) \cdot R(\text{freq}) \cdot S(\text{freq})$$

$$\log S_{21_7}(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{\text{load}} \cdot 50}{\text{ohm}} \right)^{0.5}}{U(\text{freq})_{0,0} \cdot 50 + U(\text{freq})_{0,1} + \frac{50 \cdot R_{\text{load}}}{\text{ohm}} \cdot U(\text{freq})_{1,0} + U(\text{freq})_{1,1} \cdot \frac{R_{\text{load}}}{\text{ohm}}} \right], 10$$

$$\log S_{21_8}(\text{freq}) := 10 \log \left[\frac{2 \left(\frac{R_{\text{load}} \cdot 50}{\text{ohm}} \right)^{0.5}}{V(\text{freq})_{0,0} \cdot 50 + V(\text{freq})_{0,1} + \frac{50 \cdot R_{\text{load}}}{\text{ohm}} \cdot V(\text{freq})_{1,0} + V(\text{freq})_{1,1} \cdot \frac{R_{\text{load}}}{\text{ohm}}} \right], 10$$

$$\text{Pout_real_7_dBm} := 10 \cdot \log \left(\frac{\text{Pout_real}}{\text{mW}} \right) + \log S_{21_7}(\text{freq}) - \text{T1dB}$$

$$\text{Pout_real_8_dBm} := 10 \cdot \log \left(\frac{\text{Pout_real}}{\text{mW}} \right) + \log S_{21_8}(\text{freq}) - \text{T1dB}$$



Pout_real_7_dBm = 13.341

Pout_real_8_dBm = 13.268

H2_7 := H2 + logS21_7(2freq) - T2dB

H2_7 = -33.504

H2_8 := H2 + logS21_8(2freq) - T2dB

H2_8 = -50.056

H3_7 := H3 + logS21_7(3freq) - T3dB

H3_7 = -57.856

H3_8 := H3 + logS21_8(3freq) - T3dB

H3_8 = -55.217

H4_7 := H4 + logS21_7(4freq) - T4dB

H4_7 = -70.757

H4_8 := H4 + logS21_8(4freq) - T4dB

H4_8 = -64.318

H5_7 := H5 + logS21_7(5freq) - T5dB

H5_7 = -83.784

H5_8 := H5 + logS21_8(5freq) - T5dB

H5_8 = -75.131

H6_7 := H6 + logS21_7(6freq) - T6dB

H6_7 = -96.191

H6_8 := H6 + logS21_8(6freq) - T6dB

H6_8 = -86.333

H7_7 := H7 + logS21_7(7freq) - T7dB

H7_7 = -101.838

H7_8 := H7 + logS21_8(7freq) - T7dB

H7_8 = -91.443

L1 = 180nH C1 = 1pF

Icc = 10.632mA

L1 = 180nH C1 = 1pF

Icc = 10.632mA

C2 = 4.699pF

C2 = 4.699pF

L2 = 50.473nH

L2 = 50.473nH

Lseries_modified_1 = 46.502 + 7.822i nH

Lseries_modified_1 = 46.502 + 7.822i nH

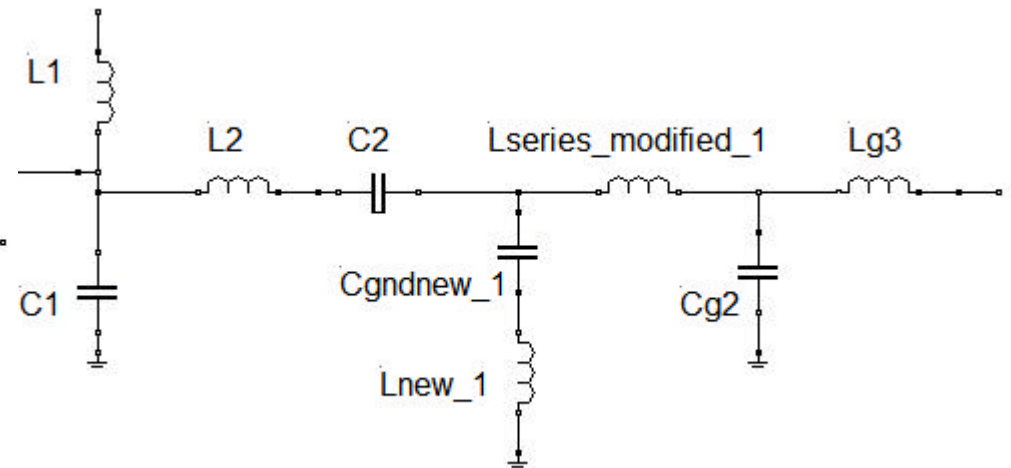
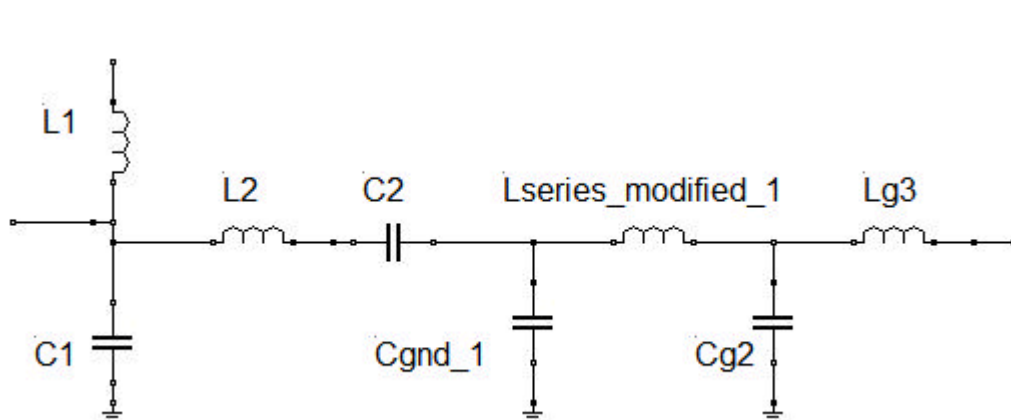
Cg2 = 4.893pF Lg3 = 46.502 nH

Cg2 = 4.893pF Lg3 = 46.502 nH

Cgnd_1 = 3.825i pF

Cgndnew_1 = 2.732i pF

Lnew_1 = -12.305i nH

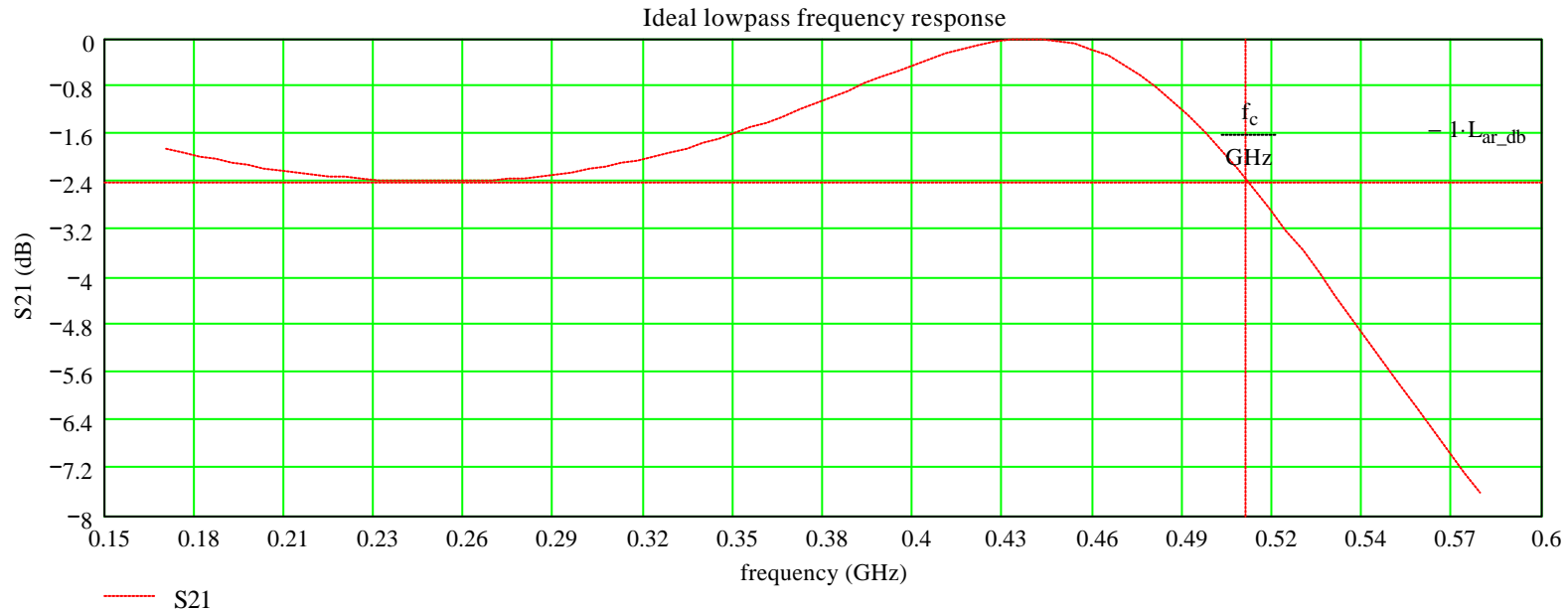


LPF Frequency Response and for Chebychev Polynomials

This subroutine is to calculate the Tchebychev polynomials for a third order filter that can be used after the tuned network to futher attenuate the harmonincs. This filter is only used if R<50, as it requires three components with the first is merged with the final stage output capacitor. When R>50, we only have two stages and so the butterworth coefficients are used 1.414, 1.414.

$$N := 3 \quad \text{order of the filter} \quad f_c := \text{freq} \cdot \text{FR} \quad f_{lp_hp_sweep_narrow} := \frac{f_c}{3}, \frac{f_c}{3} + \frac{f_c}{100} \dots f_c \cdot 1.15 \quad \epsilon := 10^{\frac{L_{ar_db}}{10}} - 1$$

$$L_A(f, f_1) := \begin{cases} 10 \cdot \log \left[1 + \epsilon \cdot \left[\cos \left(\left(N \cdot \arccos \left(\frac{f}{f_1} \right) \right) \right) \right]^2 \right] & \text{if } f \leq f_1 \\ 10 \cdot \log \left[1 + \epsilon \cdot \left[\cosh \left(\left(N \cdot \operatorname{acosh} \left(\frac{f}{f_1} \right) \right) \right) \right]^2 \right] & \text{if } f > f_1 \end{cases}$$



Calculate the g Polynomials

$$k := 1..N \quad \beta := \ln\left(\coth\left(\frac{L_{ar_db}}{17.37}\right)\right) \quad \gamma := \sinh\left(\frac{\beta}{2 \cdot N}\right) \quad a_k := \sin\left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot N}\right] \quad b_k := \gamma^2 + \left(\sin\left(\frac{k \cdot \pi}{N}\right)\right)^2 \quad g_k := 0 \quad g_0 := 1 \quad g_{N+1} := 1$$

$$g_k := \begin{cases} \frac{2 \cdot a_1}{\gamma} & \text{if } k = 1 \\ \frac{4 \cdot a_{k-1} \cdot a_k}{b_{k-1} \cdot g_{k-1}} & \text{otherwise} \end{cases}$$

$$g_1 = 2.967$$

$$g_2 = 0.78$$

$$g_3 = 2.967$$

$g(0)$ and $g(N+1)$ represent the input/output coupling for odd order filters, these are 1 representing the generator (and equal) load resistance