## Impedance Transformer

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This sheet is used to design an impedance transformer that is very small for its performance. The equations are from Ang, Lee, Leong, "Broadband Quarter-Wavelength Impedance Transformer, IEEE MTT Dec 2004.

Yellow is user input, Green is output



$$\begin{aligned} & \text{Y11}(\text{Yoo}, \text{Yoe}, \text{Yo}, \textbf{?}) \coloneqq -j \cdot \left[ \frac{2 \cdot \text{Yoo} \cdot \text{Yoe} \cdot \cos(\textbf{?})}{(\text{Yoo} + \text{Yoe}) \cdot \sin(\textbf{?})} + \frac{(\text{Yoo} - \text{Yoe})^2 \cdot \text{Yo} \cdot \sin(\textbf{?}) \cdot \cos(\textbf{?})}{\left[ (2 \cdot \text{Yo} + \text{Yoo} + \text{Yoe}) \cdot (\sin(\textbf{?}))^2 - 2\text{Yo} \right] (\text{Yoo} + \text{Yoe})} \right] \\ & \text{Y12}(\text{Yoo}, \text{Yoe}, \text{Yo}, \textbf{?}) \coloneqq -j \cdot \left[ \frac{2 \cdot \text{Yoo} \cdot \text{Yoe}}{(\text{Yoo} + \text{Yoe}) \cdot \sin(\textbf{?})} + \frac{(\text{Yoo} - \text{Yoe}) \cdot \text{Yo} \cdot \sin(\textbf{?})}{\left[ (2 \cdot \text{Yo} + \text{Yoo} + \text{Yoe}) \cdot (\sin(\textbf{?}))^2 - 2\text{Yo} \right]} \right] \\ & \text{Y21}(\text{Yoo}, \text{Yoe}, \text{Yo}, \textbf{?}) \coloneqq -j \cdot \left[ \frac{2 \cdot \text{Yoo} \cdot \text{Yoe}}{(\text{Yoo} + \text{Yoe}) \cdot \sin(\textbf{?})} + \frac{(\text{Yoo} - \text{Yoe}) \cdot \text{Yo} \cdot \sin(\textbf{?})}{\left[ (2 \cdot \text{Yo} + \text{Yoo} + \text{Yoe}) \cdot (\sin(\textbf{?}))^2 - 2\text{Yo} \right]} \right] \\ & \text{Y22}(\text{Yoo}, \text{Yoe}, \text{Yo}, \textbf{?}) \coloneqq -j \cdot \left[ \frac{2 \cdot \text{Yoo} \cdot \text{Yoe} \cdot \cos(\textbf{?})}{(\text{Yoo} + \text{Yoe}) \cdot \sin(\textbf{?})} + \frac{(2 \cdot \text{Yo} + \text{Yoo} + \text{Yoe}) \cdot (\sin(\textbf{?}))^2 - 2\text{Yo} \right]}{\left[ (2 \cdot \text{Yo} + \text{Yoo} + \text{Yoe}) \cdot (\sin(\textbf{?}))^2 - 2\text{Yo} \right]} \right] \end{aligned}$$

$$\operatorname{Yin}(\operatorname{Zoo}, \operatorname{Zoe}, \operatorname{Zo}, \operatorname{Zo}, \operatorname{Z2}, \mathbf{?}) \coloneqq \left(\operatorname{Y1l}\left(\frac{1}{\operatorname{Zoo}}, \frac{1}{\operatorname{Zoe}}, \frac{1}{\operatorname{Zo}}, \mathbf{?}\right) - \frac{\operatorname{Y12}\left(\frac{1}{\operatorname{Zoo}}, \frac{1}{\operatorname{Zoe}}, \frac{1}{\operatorname{Zo}}, \mathbf{?}\right) \operatorname{Y2l}\left(\frac{1}{\operatorname{Zoo}}, \frac{1}{\operatorname{Zoe}}, \frac{1}{\operatorname{Zo}}, \mathbf{?}\right)}{\operatorname{Y22}\left(\frac{1}{\operatorname{Zoo}}, \frac{1}{\operatorname{Zoe}}, \frac{1}{\operatorname{Zo}}, \mathbf{?}\right) + \frac{1}{\operatorname{Z2}}}\right)$$

## **Calculations**

The calculations are done using the Find command where convergences is helped by setting CTOL=0.05. After convergence, put the the output results back to the guess values and repeat the convergence until the two are the same, then change CTOL to 0.0005 and converge again.

This sheet is setup so that the calculate button F9 needs to be pressed.

guess	$Zo := 154.3 \cdot \mathbf{O}$	
	Zoe := 153.9 · <b>O</b>	
	Zoo := 39.8 · <b>O</b>	
Given		$Z1 := 50 \cdot \mathbf{O}$ $Z2 := 100 \cdot \mathbf{O}$
$Yin(Zoo, Zoe, Zo, Z2, 90deg) = \frac{1}{Z1}$		Find(Zo, Zoe, Zoo) = $\begin{pmatrix} 154.3 \\ 153.9 \\ 39.8 \end{pmatrix}$ · <b>O</b>
guess	$Z_{\mathbf{Q}} := 205.2 \cdot \mathbf{O}$	
	$Z_{\text{MOR}} := 233.4 \cdot \mathbf{O}$	
	$Z_{00} := 54.0 \cdot 0$	
Given		$Z_1 = 50  \mathbf{O}$ $Z_2 = 200  \mathbf{O}$
Yi	$n(Zoo, Zoe, Zo, Z2, 90deg) = \frac{1}{Z1}$	Find(Zo,Zoe,Zoo) = $\begin{pmatrix} 205.2\\233.4\\54 \end{pmatrix}$ · <b>O</b>
guess	7	
C	$Z_{00} := 258.2 \cdot 0$	
	$Z_{000} := 80.0 \cdot 0$	
Given		$\underline{Z1} := 50 \cdot \mathbf{O}$
Yi	$n(Zoo, Zoe, Zo, Z2, 90deg) = \frac{1}{Z1}$	$\operatorname{Find}(\operatorname{Zo}, \operatorname{Zoe}, \operatorname{Zoo}) = \begin{pmatrix} 258.3 \\ 233.5 \\ 75.5 \end{pmatrix} \cdot \mathbf{O}$

## **Simulations**

50 Ohms to 100 Ohms, Zo=154.3, Zoe=153.9, Zo=39.8 50 Ohms to 200 Ohms, Zo=205.2, Zoe=233.4, Zo=54.0

Please refer to other MathCAD sheets on this site to convert from Zo and Zoo/Zoe to microstrip widths and spacings.



