

Error Function Curves for different Modulation Schemes

This sheet plots the error function curves to calculate probability of error (P_e) for different E_b/N_0 , only for FM, PM, or QAM modulation.

$x := -5, -4.9.. 25$

Range of E_b/N_0 in dB

$$s(x) := 10^{\frac{x}{10}}$$

$$\text{erfc}(x) := 1 - \text{erf}(x)$$

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$$P_{\text{psk}}(x) := 0.5 \text{erfc}(\sqrt{s(x)})$$

BER of Coherent Binary PSK

$$P_{\text{fsk}}(x) := 0.5 \text{erfc}\left(\sqrt{\frac{s(x)}{2}}\right)$$

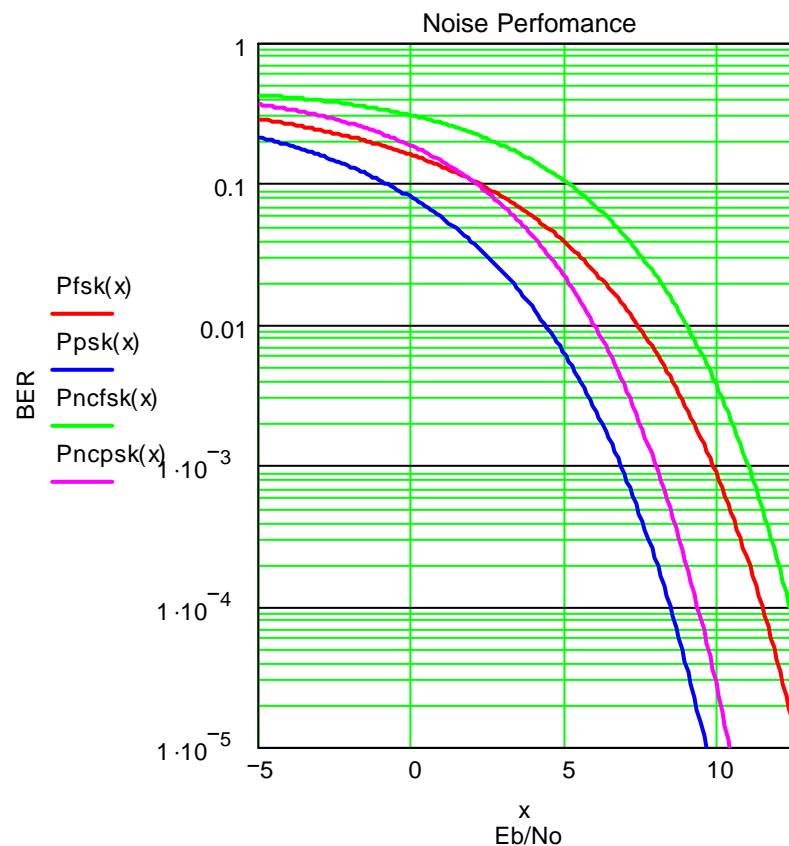
BER of Coherent Binary FSK

$$P_{\text{ncfsk}}(x) := 0.5 \cdot \left(\exp\left(\frac{-s(x)}{2}\right) \right)$$

BER of Non-Coherent Binary FSK

$$P_{\text{ncpsk}}(x) := 0.5 \cdot (\exp(-s(x)))$$

BER of Non-Coherent Differential PSK. A 0 is sent by +180 degrees, a 1 is sent by no change



M-Ary FSK

In multi-level FSK the minimum frequency spacing is $1/2T$ Hertz.

For example with 2-fsk a symbol rate of 8000 symbols per second, corresponds to 8000 bits per second and requires the frequency spacing is 4kHz. The bandwidth required for low adjacent channel power is approx 12kHz.

With 4-fsk in the same bandwidth, the symbol rate is 2000 symbols per second corresponding to 8000 bits per second and a frequency spacing of $1\text{kHz} \cdot 4 = 4\text{kHz}$. This has lower spectral regrowth in the adjacent channel at the cost of lower BER, higher complexity on Tx & Rx and more stable frequency reference.

$$P_{\text{fc4}}(x) := 0.5 \cdot (4 - 1) \cdot \text{erfc} \left(\sqrt{\frac{s(x)}{2}} \right)$$

BER of Coherent 4-level FSK

$$P_{\text{fn4}}(x) := 0.5 \cdot (4 - 1) \cdot \exp \left(-\frac{s(x)}{2} \right)$$

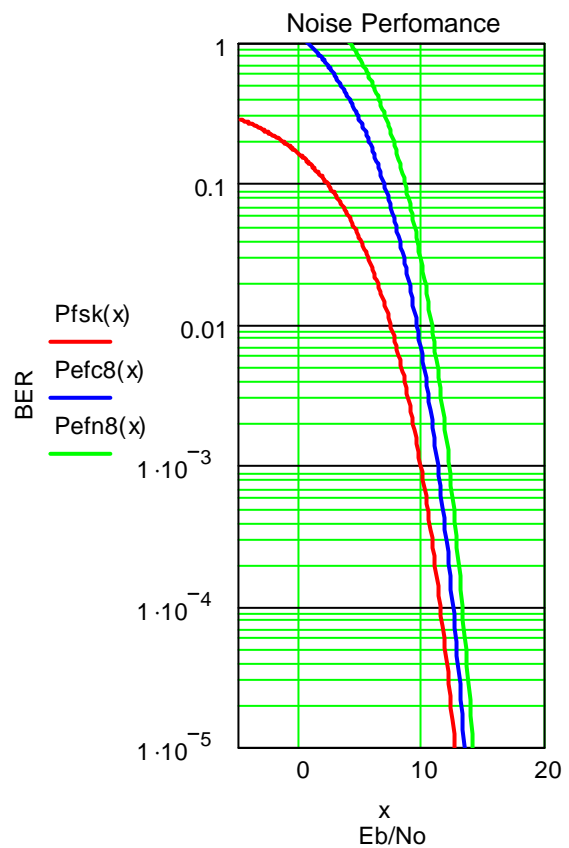
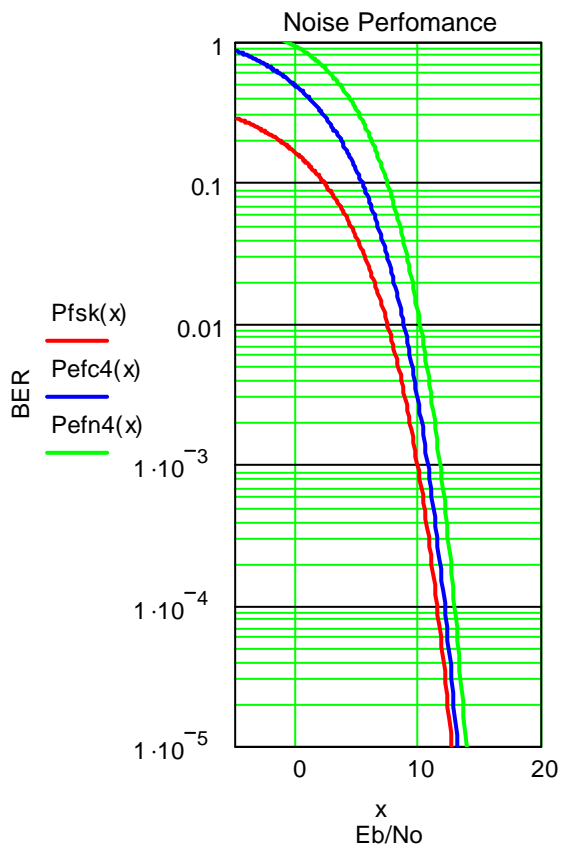
BER of Non-Coherent 4-level FSK

$$P_{\text{fc8}}(x) := 0.5 \cdot (8 - 1) \cdot \text{erfc} \left(\sqrt{\frac{s(x)}{2}} \right)$$

BER of Coherent 8-level FSK

$$P_{\text{fn8}}(x) := 0.5 \cdot (8 - 1) \cdot \exp \left(-\frac{s(x)}{2} \right)$$

BER of Non-Coherent 8-level FSK



M-Ary PSK

$$P_{\text{qpsk}}(x) := 0.5 \operatorname{erfc}(\sqrt{s(x)})$$

Coherent QPSK This is the same as P_{epc4}

$$P_{\text{epc4}}(x) := \operatorname{erfc}\left(\sqrt{s(x)} \cdot \sin\left(\frac{\pi}{4}\right)\right)$$

BER of Coherent 4-level PSK

$$P_{\text{epn4}}(x) := \operatorname{erfc}\left(\sqrt{2 \cdot s(x)} \cdot \sin\left(\frac{\pi}{2 \cdot 4}\right)\right)$$

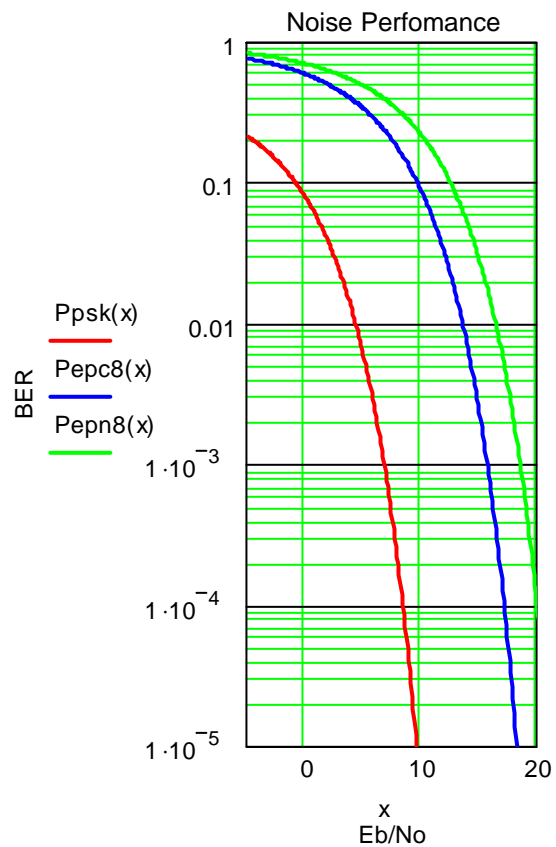
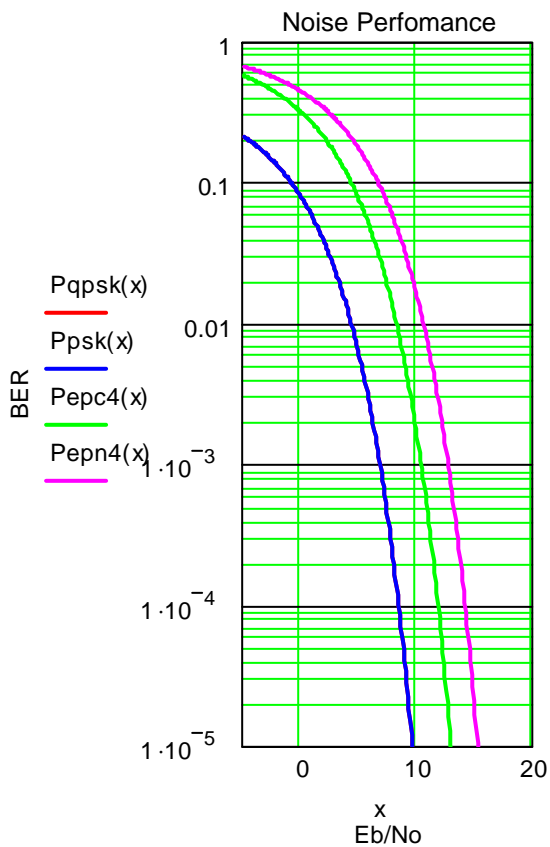
BER of Non-Coherent 4-level PSK

$$P_{\text{epc8}}(x) := \operatorname{erfc}\left(\sqrt{s(x)} \cdot \sin\left(\frac{\pi}{8}\right)\right)$$

BER of Coherent 8-level PSK

$$P_{\text{epn8}}(x) := \operatorname{erfc}\left(\sqrt{2 \cdot s(x)} \cdot \sin\left(\frac{\pi}{2 \cdot 8}\right)\right)$$

BER of Non-Coherent 8-level PSK



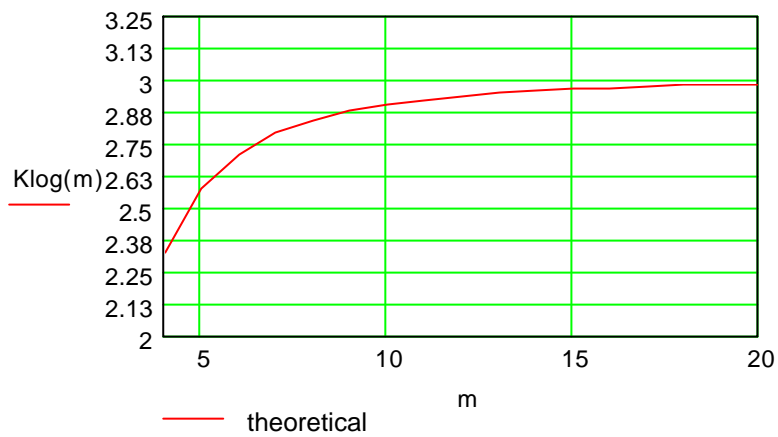
Coherent Vs Non Coherent M-Ary PSK

In multi-level PSK the same BER performance for non-coherent detection compared to coherent by increasing the energy per symbol. The power increase levels off to a 3dB power increase.

$m := 4.. 20$

$$k(m) := \left[\frac{\left(\sin\left(\frac{\pi}{m}\right) \right)^2}{2 \cdot \left(\sin\left(\frac{\pi}{2 \cdot m}\right) \right)^2} \right]$$

$$Klog(m) := 10 \cdot \log(k(m))$$



M-Ary QAM

The QAM signal is shown here in terms of E_b/N_0 . In other words the energy per bit is constant, all that is changing is the noise window is getting smaller. As the number of levels increase then so does the transmitted power.

$$Peqc4(x) := 2 \cdot \left(1 - \frac{1}{\sqrt{4}}\right) \cdot \text{erfc}(\sqrt{s(x)})$$

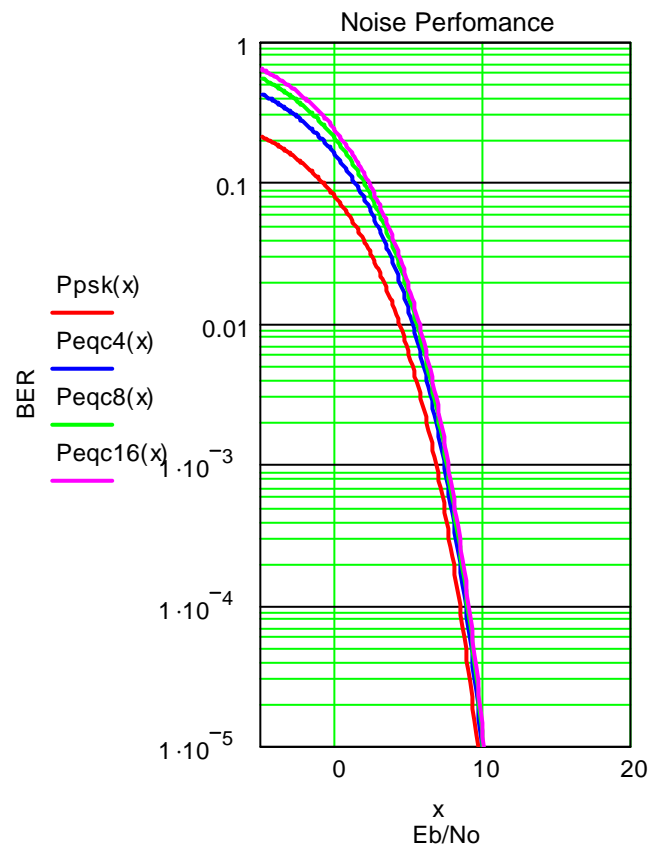
BER of Coherent 4-level QAM

$$Peqc8(x) := 2 \cdot \left(1 - \frac{1}{\sqrt{8}}\right) \cdot \text{erfc}(\sqrt{s(x)})$$

BER of Coherent 8-level QAM

$$Peqc16(x) := 2 \cdot \left(1 - \frac{1}{\sqrt{16}}\right) \cdot \text{erfc}(\sqrt{s(x)})$$

BER of Coherent 16-level QAM



M-Ary QAM

The QAM signal is shown here in terms of E_{av}/N_0 . In other words the total power transmitted is kept constant. $s(x)$ in this section = E_{av}/N_0 .

$$P_{avqc4}(x) := 2 \cdot \left(1 - \frac{1}{\sqrt{4}}\right) \cdot \operatorname{erfc} \left[\sqrt{\frac{s(x) \cdot 3}{2 \cdot (4 - 1)}} \right] \quad \text{BER of Coherent 4-level QAM}$$

$$P_{avqc8}(x) := 2 \cdot \left(1 - \frac{1}{\sqrt{8}}\right) \cdot \operatorname{erfc} \left[\sqrt{\frac{s(x) \cdot 3}{2 \cdot (8 - 1)}} \right] \quad \text{BER of Coherent 8-level QAM}$$

$$P_{avqc16}(x) := 2 \cdot \left(1 - \frac{1}{\sqrt{16}}\right) \cdot \operatorname{erfc} \left[\sqrt{\frac{s(x) \cdot 3}{2 \cdot (16 - 1)}} \right] \quad \text{BER of Coherent 16-level QAM}$$

