Inductance Calulation of Different Shapes

This sheet is used to design planar spiral inductors used on RFICs.

The expressions are from this paper in the IEEE Journal of Solid State Circuits, 10th October 1999, pages 1419-1424. Paper attached. This paper presents several equations, two are caculated here, the **modified Wheeler** and a **Current Sheet Approximation**. These are calculated for the two common planar inductor types and shown with measured results

The main advantage of this is a starting point for a field solver.

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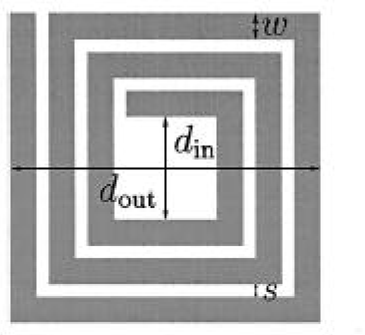
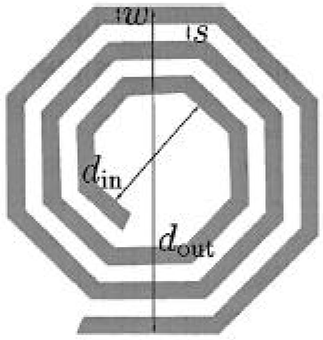
crobat Docume



(a)



(b)

The range of dout is from 100 to 480um; w is from 2 to 0.3*dout; s is from 2um to 3*w; din is from 0.1 to 0.9*dout. Total inductance from 0.5 to 100nH

$\mathbf{\mu}\mathrm{m} := \frac{\mathrm{m}}{10^{6}}$ $\mathbf{\mu}_0 := 4\cdot\mathbf{p}\cdot10^{-7}\cdot\frac{\mathrm{newton}}{\mathrm{amp}^{2}}$ $\mathrm{nH} := \frac{\mathrm{H}}{10^{9}}$ $\mathrm{mOhm} := \frac{\mathrm{ohm}}{10^{3}}$

$\mathrm{freq} := 1\times10^{9}\cdot\mathrm{Hz}$

$\mathrm{resistivity} := 10\cdot\mathrm{mOhm}$

$$d_{in}(d_{out},n,w,s) := d_{out} - 2\cdot w - \mathrm{halfturn}(n)\cdot(w+s)$$

$$\mathrm{halfturn}(n) := \mathrm{floor}[2(n-0.25)] - 1$$

$$d_{avg}(d_{out},n,w,s) := \frac{d_{in}(d_{out},n,w,s) + d_{out}}{2}$$

$$?(d_{out},n,w,s) := \frac{d_{out} - d_{in}(d_{out},n,w,s)}{d_{out} + d_{in}(d_{out},n,w,s)}$$

$d_{in}(11\mathbf{\mu}\mathrm{m}, 2.25, 1\mathbf{\mu}\mathrm{m}, 1\mathbf{\mu}\mathrm{m}) = 3\cdot\mathbf{\mu}\mathrm{m}$

$d_{in}(300\mathbf{\mu}\mathrm{m}, 2.25, 20\mathbf{\mu}\mathrm{m}, 10\mathbf{\mu}\mathrm{m}) = 170\cdot\mathbf{\mu}\mathrm{m}$

$?(321\mathbf{\mu}\mathrm{m}, 3.75, 16.5\mathbf{\mu}\mathrm{m}, 1.9\mathbf{\mu}\mathrm{m}) = 0.288$

Approximate Resistance

$$\mathrm{resistance}(d_{out},n,w,s) := \frac{d_{avg}(d_{out},n,w,s)\cdot\mathbf{p}\cdot\mathrm{resistivity}}{w}$$

The resistance is calculated as simply the average circumference x turns x resistivity. This is very crude and ignores corner affects, but will try to allow Q to be plotted..

$\mathrm{resistance}(339\mathbf{\mu}\mathrm{m}, 3.75, 10\mathbf{\mu}\mathrm{m}, 1.9\mathbf{\mu}\mathrm{m}) = 0.921\cdot\mathrm{ohm}$

Square inductor (a) Calculation

$K_{a1} := 2.34$ $K_{a2} := 2.75$

$$L_{smw}(d_{out}, n, w, s) := K_{a1} \cdot \mu_0 \cdot \frac{n^2 d_{avg}(d_{out}, n, w, s)}{1 + K_{a2} \cdot ?(d_{out}, n, w, s)}$$

Modified wheeler equation for three square inductors compared to measured values

$L_{smw}(321\mu m, 3.75, 16.5\mu m, 1.9\mu m) = 5.76 \cdot nH$ #10, measured 6.1nH

$L_{smw}(339\mu m, 5.75, 10\mu m, 1.9\mu m) = 15.33 \cdot nH$ #15, measured 16.2nH

$L_{smw}(400\mu m, 3.75, 31.6\mu m, 1.9\mu m) = 4.7 \cdot nH$ #14, measured 4.9nH

$C_{a1} := 1.27$ $C_{a2} := 2.07$ $C_{a3} := 0.18$ $C_{a4} := 0.13$

$$L_{scsa}(d_{out}, n, w, s) := \frac{\mu_0 \cdot n^2 \cdot d_{avg}(d_{out}, n, w, s) C_{a1}}{2} \cdot \left(\ln\left(\frac{C_{a2}}{?(d_{out}, n, w, s)} \right) + C_{a3} \cdot ?(d_{out}, n, w, s) + C_{a4} \cdot ?(d_{out}, n, w, s)^2 \right)$$

Current Sheet aproximation for three square inductors compared to measured values

$L_{scsa}(321\mu m, 3.75, 16.5\mu m, 1.9\mu m) = 5.7 \cdot nH$ #10, measured 6.1nH

$L_{scsa}(339\mu m, 5.75, 10\mu m, 1.9\mu m) = 15.2 \cdot nH$ #15, measured 16.2nH

$L_{scsa}(400\mu m, 3.75, 31.6\mu m, 1.9\mu m) = 4.67 \cdot nH$ #14, measured 4.9nH

Hexagon inductor (c) Calculation

$K_{c1} := 2.33$ $K_{c2} := 3.82$

$$L_{hmw}(d_{out}, n, w, s) := K_{c1} \cdot \mu_0 \cdot \frac{n^2 d_{avg}(d_{out}, n, w, s)}{1 + K_{c2} \cdot ?(d_{out}, n, w, s)}$$

Modified wheeler equation for two hex inductors compared to measured values

$L_{hmw}(346\mu m, 4, 18\mu m, 2\mu m) = 5.95 \cdot nH$ #55, measured 7.5nH

$L_{hmw}(326\mu m, 5, 8\mu m, 12\mu m) = 7.22 \cdot nH$ #58, measured 7.2nH

$C_{c1} := 1.09$ $C_{c2} := 2.23$ $C_{c3} := 0.0$ $C_{c4} := 0.17$

$$L_{hcsa}(d_{out}, n, w, s) := \frac{\mu_0 \cdot n^2 \cdot d_{avg}(d_{out}, n, w, s) C_{c1}}{2} \cdot \left(\ln\left(\frac{C_{c2}}{?(d_{out}, n, w, s)} \right) + C_{c3} \cdot ?(d_{out}, n, w, s) + C_{c4} \cdot ?(d_{out}, n, w, s)^2 \right)$$

Current Sheet aproximation for two hex inductors compared to measured values

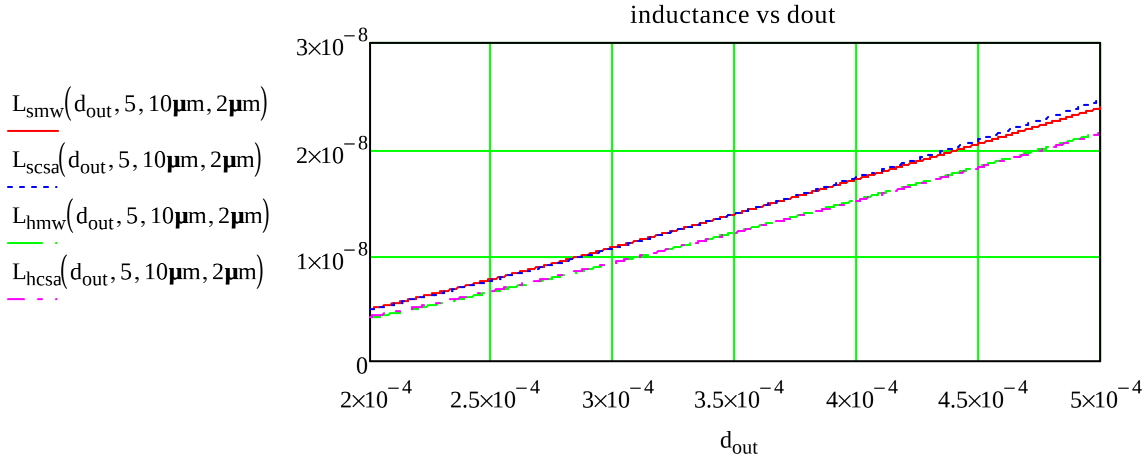
$L_{hcsa}(346\mu m, 4, 18\mu m, 2\mu m) = 6.02 \cdot nH$ #55, measured 7.5nH

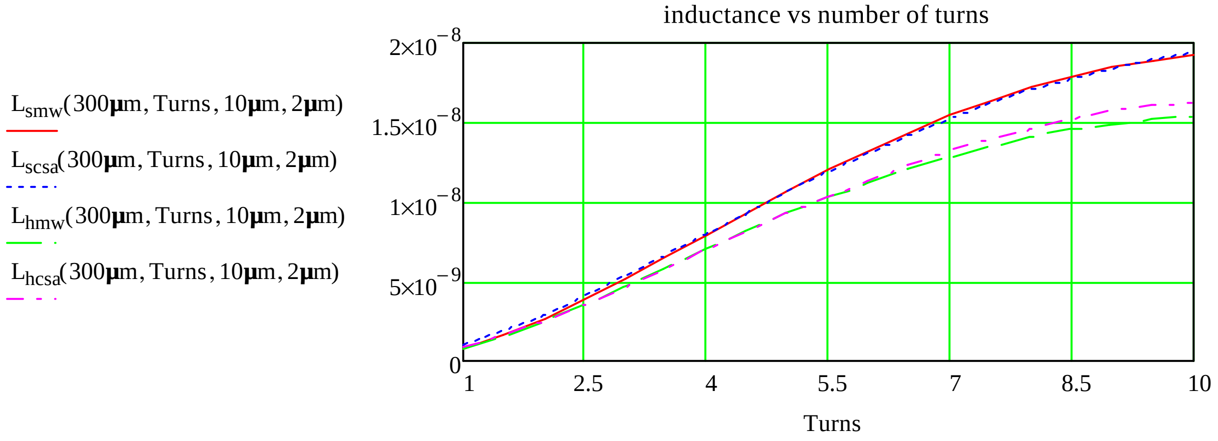
$L_{hcsa}(326\mu m, 5, 8\mu m, 12\mu m) = 7.42 \cdot nH$ #58, measured 7.2nH

Plots of inductors

$Turns := 1, 1.5 .. 10$

The inductance increases more with outside diameter presumably because the tracks length is a square of this.





The Q is not correct and should be about 20. This is because the conducted losses are only part of the story and the induced losses into the substrate are more important and not included.