

Power Amplifier Harmonic Calculator

This page calculates the harmonic content out of FET Power Amplifiers using a Fourier Analysis of the output waveform. Class C amplifiers are more nonlinear than class A and produce a greater number of harmonics. This sheet calculates the harmonic level & efficiency. The transfer function of a FET PA transistor is approximated to a square law above a certain threshold.

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<http://www.sunshadow.co.uk/chris.htm>

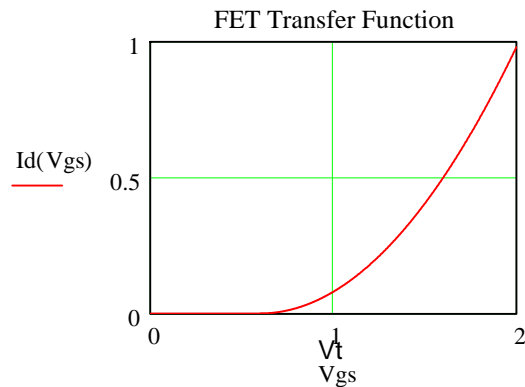
The equations used here are from class notes produced by **Jens Vidkaer** at Aalborg University, copy attached ->.

The value used for gm used here is when the FET is in the linear (Velocity Saturation) portion of the Id to Vgs curve. Vt

This is not a bad starting point but the equation for Id is a bit more complex.

The input signal is assumed sinusoidal.
 Vpp is the input peak-peak voltage range,
 Vb is the bias voltage.
 θ is the conduction angle.

Ip is the peak output current for this input.
 gm and Vpp are only used in the first section, and then cancel out.



$$V_t := 0.6V \quad g_m := 500 \cdot \frac{\text{mA}}{V} \quad V_{pp} := 1 \cdot \text{volt}$$

$$I_d(V_{gs}) := \begin{cases} 0 & \text{if } V_{gs} < V_t \\ g_m (V_{gs} - V_t)^2 & \text{otherwise} \end{cases}$$

$$\theta(V_b, V_{pp}) := 2 \cdot \arccos \left[\frac{V_t - V_b}{\left(\frac{V_{pp}}{2} \right)} \right]$$

The first step is to reproduce the plots shows in the reference document --->.

$$y_p(\theta) := gm \cdot \left(\frac{V_{pp}}{2} \right)^2 \cdot \left(1 - \cos\left(\frac{\theta \cdot \text{deg}}{2} \right) \right)^2 \cdot \text{volt}^{-1}$$

$$y_0(\theta) := \frac{gm}{\pi} \cdot \left(\frac{V_{pp}}{2} \right)^2 \cdot \left(\frac{\theta \cdot \text{deg}}{2} - \frac{3}{4} \cdot \sin(\theta \cdot \text{deg}) + \frac{\theta \cdot \text{deg}}{4} \cdot \cos(\theta \cdot \text{deg}) \right) \cdot V^{-1}$$

$$y_1(\theta) := \frac{gm}{\pi} \cdot \left(\frac{V_{pp}}{2} \right)^2 \cdot \left(\frac{3}{2} \cdot \sin\left(\frac{\theta \cdot \text{deg}}{2} \right) + \frac{1}{6} \cdot \sin\left(\frac{3}{2} \cdot \theta \cdot \text{deg} \right) - \theta \cdot \text{deg} \cdot \cos\left(\frac{\theta \cdot \text{deg}}{2} \right) \right) \cdot V^{-1}$$

$$y_2(\theta) := \frac{gm}{\pi} \cdot \left(\frac{V_{pp}}{2} \right)^2 \cdot \left(\frac{\theta \cdot \text{deg}}{4} - \frac{1}{3} \cdot \sin(\theta \cdot \text{deg}) + \frac{1}{24} \cdot \sin(2\theta \cdot \text{deg}) \right) \cdot V^{-1}$$

y_p is peak consumption

y_0 is the dc power

y_1 is the wanted power, i.e. the first harmonic

y_2 is the second harmonic.

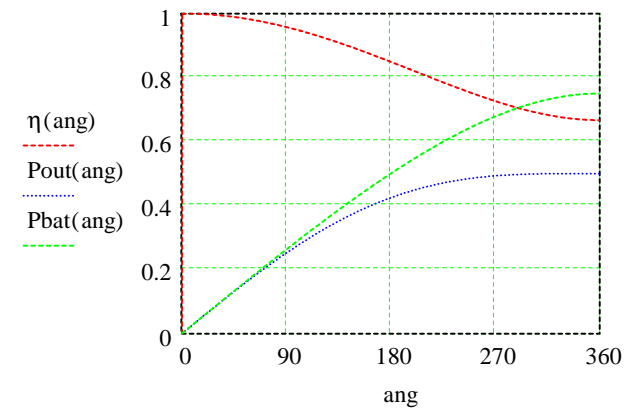
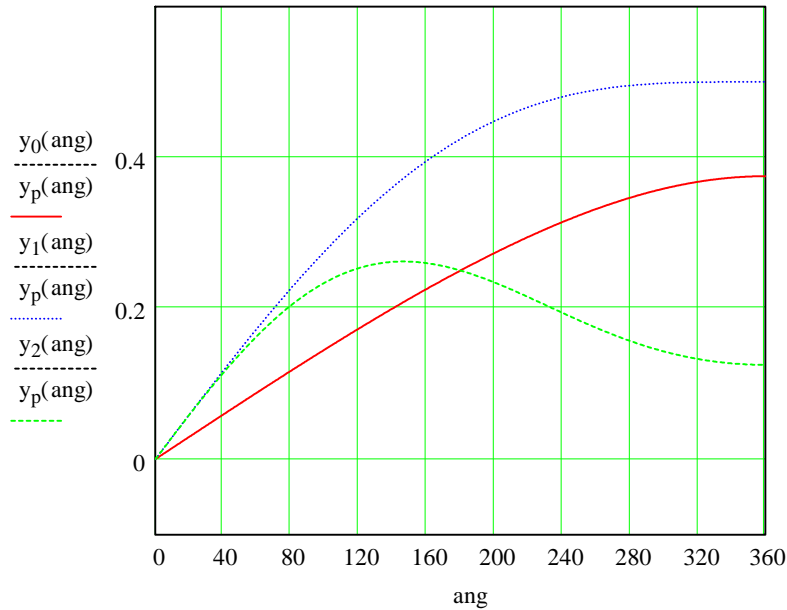
I could not get the equations for y_3 and above to agree with the reference document and are omitted.

The efficiency is the 'output power'/'battery power'

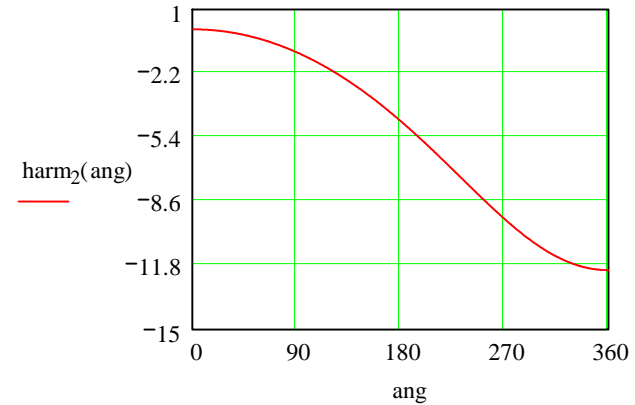
The output power is P_{out}

The battery power is P_{bat}

$$\text{harm}_2(\theta) := 20 \cdot \log\left(\left| \frac{y_2(\theta)}{y_1(\theta)} \right| \right) \quad \eta(\theta) := \frac{y_1(\theta)}{2y_0(\theta)} \quad P_{out}(\theta) := \frac{y_1(\theta)}{y_p(\theta)} \quad P_{bat}(\theta) := 2 \cdot \frac{y_0(\theta)}{y_p(\theta)}$$



Now we can calculate the level of second harmonic compared to the wanted signal for different conduction angles



$$y_p(V_b, V_{pp}) := gm \cdot \left(\frac{V_{pp}}{2} \right)^2 \cdot \left(1 - \cos\left(\frac{\theta(V_b, V_{pp})}{2} \right) \right)^2 \cdot \text{volt}^{-1}$$

The next step is to carry out a similar analysis but this time to change the bias voltage V_b , and the peak to peak input voltage V_{pp} .

$$y_0(V_b, V_{pp}) := \frac{gm}{\pi} \left(\frac{V_{pp}}{2} \right)^2 \cdot \left(\frac{\theta(V_b, V_{pp})}{2} - \frac{3}{4} \cdot \sin(\theta(V_b, V_{pp})) + \frac{\theta(V_b, V_{pp})}{4} \cdot \cos(\theta(V_b, V_{pp})) \right) \cdot V^{-1}$$

The threshold voltage V_T for the FET is set above.

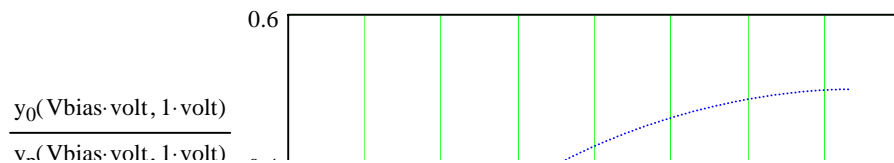
$$y_1(V_b, V_{pp}) := \frac{gm}{\pi} \cdot \left(\frac{V_{pp}}{2} \right)^2 \cdot \left(\frac{3}{2} \cdot \sin\left(\frac{\theta(V_b, V_{pp})}{2} \right) + \frac{1}{6} \cdot \sin\left(\frac{3}{2} \cdot \theta(V_b, V_{pp}) \right) - \theta(V_b, V_{pp}) \cdot \cos\left(\frac{\theta(V_b, V_{pp})}{2} \right) \right) \cdot V^{-1}$$

$$y_2(V_b, V_{pp}) := \frac{gm}{\pi} \cdot \left(\frac{V_{pp}}{2} \right)^2 \cdot \left(\frac{\theta(V_b, V_{pp})}{4} - \frac{1}{3} \cdot \sin(\theta(V_b, V_{pp})) + \frac{1}{24} \cdot \sin(2\theta(V_b, V_{pp})) \right) \cdot V^{-1}$$

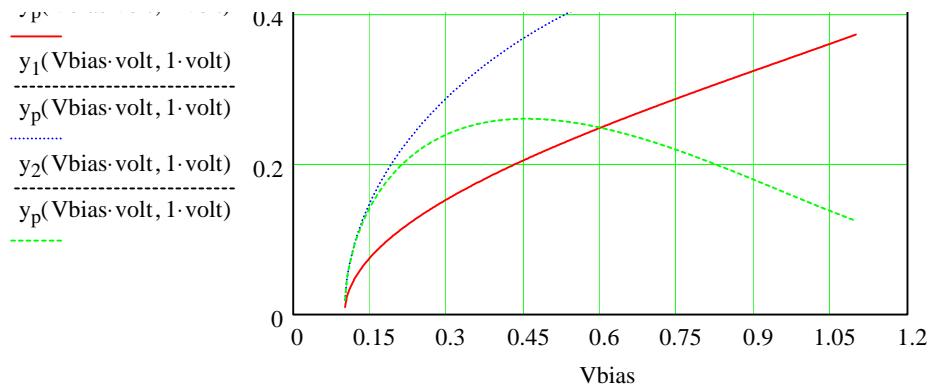
$$\eta(V_b, V_{pp}) := \frac{y_1(V_b, V_{pp})}{2 y_0(V_b, V_{pp})}$$

$$P_{bat}(V_b, V_{pp}) := 2 \cdot \frac{y_0(V_b, V_{pp})}{y_p(V_b, V_{pp})}$$

$$P_{out}(V_b, V_{pp}) := \frac{y_1(V_b, V_{pp})}{y_p(V_b, V_{pp})}$$



$$\text{harm}_2(V_b, V_{pp}) := 20 \cdot \log \left(\left| \frac{y_2(V_b, V_{pp})}{y_1(V_b, V_{pp})} \right| \right)$$



Now we can calculate the level of second harmonic compared to the wanted signal as either the bias or peak to peak voltage changes.

Not as far down as I would like, and that is the point of this exercise. The square law of the FET gives a very large second harmonic compared to a bipolar. The other harmonics are well down and better than a bipolar would give.

A lesson to learn!!!

